Math 101 Calculus I 12.01.2012

Izmir University of Economics Faculty of Arts and Sciences, Department of Mathematics

Final Exam

Student Name and Department: ...........................................

Section: Check for your instructor and course program below:

☐ Halil ORUC, Fri. 08:30 - 11:20, Fri. 12:30-15:20

☐ Gökhan BILHAN, Tues. 8:30-11:20, Tues. 13:30-16:20

☐ Gökhan BILHAN, Wednesday, 12:30-15:20

☐ İbrahim Çanak, Mon. 8:30-11:20, Mon. 12:30-15:20

☐ İbrahim Çanak, Thursday, 14:30-17:20

☐ Ilgin SAGER, Tuesday, 08:30-11:20, Tues. 13:30-16:20

☐ Ilgin SAGER, Wednesday, 08:30-11:20

☐ Ilgin SAGER, Thur. 08:30-11:20, Thur. 14:30-17:20

☐ Ünal UFUKTEPE, Wednesday, 08:30-11:20 Good Luck...
1. (30pts) Let \( f(x) = \frac{x^2}{(x-1)^3} \)

(a) (8) Find the intervals on which the graph of \( f \) is increasing/decreasing.
(b) (7) Evaluate critical and max/min values of \( f \).
(c) (5) Find the intervals on which the graph of \( f \) is concave up/down.
(d) (10) Sketch the graph of \( f(x) \) (show your work).

Solution:

\[ f'(x) = \frac{2x(x-1)^2 - 3(x-1)^2 x^2}{(x-1)^6} = \frac{2x(x-1) - 3x^2}{(x-1)^4} = \frac{-x(x+2)}{(x-1)^4} \]

\[ f'(x) \]

\[ \begin{array}{c|c|c|c|c}
-2 & 0 & 1 & \\
- & + & - & - \\
\end{array} \]

\( f \) is inc. on \((-2, 0)\) and dec. on \((-\infty, -2) \cup (0, 1)\).

(b) \( f'(x) = 0 \rightarrow x = 0 \) and \( x = -2 \) are critical values.
\( f(-2) = \frac{-4}{27} \) is the local min val.
\( f(0) = 0 \) is the local max val.

(c) \( f''(x) = \frac{-x^2 - 2x}{(x-1)^4} \Rightarrow f''(x) = \frac{2(x^2 + 4x + 1)}{(x-1)^5} \Rightarrow x = -2 - \sqrt{3}, x = -2 + \sqrt{3} \)

\[ f''(x) \]

\[ \begin{array}{c|c|c|c|c}
-2 - \sqrt{3} & -2 + \sqrt{3} & 1 & \\
- & + & - & - \\
\end{array} \]

(d) \( x = 1 \) v.a.
\( y = 0 \) H.A.
\( (x, y) \) x-intercepts

\[ f \]

\[ \begin{array}{c|c|c|c|c}
0 & 1 & 1 & \\
- & + & - & + \\
\end{array} \]
2. (5+5+8−7 pt.) Evaluate the followsings

(a) \( \lim_{x \to 2} \frac{x^2}{(2 + x)^4} = ? \)

(b) \( \lim_{x \to \infty} e^{-x} \ln x = ? \)

(c) \( \int \frac{\ln x^2}{x} \, dx = ? \)

(d) \( \int (3 - 7x^2)^3 2x \, dx = ? \)

Solution:

(a) \( \lim_{x \to 2} \frac{x^4}{(2 + x)^4} = \frac{16}{27} = \infty \)

(b) \( \lim_{x \to \infty} \frac{\ln x}{e^x} = \lim_{x \to \infty} \frac{\frac{1}{x}}{e^x} = \lim_{x \to \infty} \frac{1}{xe^x} = 0 \)

(c) \( \int \frac{\ln x^2}{x} \, dx = 2 \int \frac{\ln x}{x} \, dx = 2 \int u \, du = 2 \frac{u^2}{2} + C = (\ln x)^2 + C \)

(d) \( u = 3 - 7x^2 \)
\( du = -14x \, dx \)
\( \int (3 - 7x^2)^3 2x \, dx = -\frac{2}{14} \int u^3 \, du = -\frac{1}{7} \cdot \frac{u^4}{4} + C = -\frac{(3 - 7x^2)^4}{28} + C \)
3. (25 pt) A company manufactures and sells e-book readers per month. The monthly
cost and price-demand equations are, respectively
\[ C(x) = 350x + 50000 \]
\[ p = 500 - 0.025x, \ 0 \leq x \leq 20000 \]
Find the maximum revenue and profit.

Solution:

\[ R(x) = xp = x(500 - 0.025x) \]
\[ = 500x - 0.025x^2 \]
\[ R'(x) = 500 - 0.050x = 0 \]
\[ x = \frac{500}{0.050} = \frac{500000}{50} = 10000 \]  
\[ R''(x) = -0.050 < 0 \]  
So

\[ R(10000) = 50000000 - 0.025 \cdot 10000 \cdot 10000 = 50000000 - 2500000 = 27500000 \]
\[ \text{max revenue} \]

\[ P(x) = R(x) - C(x) \]
\[ = 500x - 0.025x^2 - 350x - 50000 \]
\[ P(x) = 150x - 0.025x^2 - 50000 \]
\[ P'(x) = 150 - 0.050x = 0 \]
\[ x = \frac{150}{0.050} = \frac{150000}{50} = 3000 \]  
\[ P''(x) = -0.050 < 0 \]  
So

\[ P(3000) = 150 \cdot 3000 - \frac{25}{1000} \cdot 3000 \cdot 2000 - 50000 \]
\[ = 450000 - 225000 - 50000 = 175000 \]
\[ \text{max profit} \]
4. (25 pt) In a computer assembly plant, a new employee, on the average, is able to assemble

\[ N(t) = 10(1 - e^{-0.4t}) \]

units \( t \) days of on-the-job training. What is the rate of learning after 5 days?

\[ N'(t) = 0.4 \times 10 \cdot e^{-0.4t} = 4 \cdot e^{-0.4t} \]

\[ N'(5) = 4 \cdot e^{-0.4 \times 5} = 4 \cdot e^{-2} = \frac{4}{e^2} \]

\[ = 10 \]