IUE – MATH 280 – Introduction to Probability and Statistics

First Midterm Exam — October 30, 2014 — 19:00 – 20:30

Name Surname : _____ ID # : _____

Q1	Q2	Q3	Q4	Q5	TOTAL
20	20	20	20	20	100

- The exam consists of 5 questions.
- Please read the questions carefully.
- Write your answers in the empty space at the end of each question. Be neat.
- Show all your work. Correct answers without sufficient explanation might not get full credit.
- Exchange of any material (e.g., calculators, rubbers, tables, etc.) is not allowed.
- Dictionaries and mobile phones are not allowed.
- You may use any empty space for scratch work.
- Don't forget to **indicate** your instructor and your section in the following table.

Instructor			
Guvenc Arslan	ITF	Mon 17-18 & Wed 12-15	
Femin Yalcin Gulec	BA1	Mon 18-19 & Tue 11-14	
Femin Yalcin Gulec	BA2	Tue 10-11 & Thu 16-19	
Femin Yalcin Gulec	ITF	Mon 16-17 & Wed 16-19	
Femin Yalcin Gulec	LM	Mon 17-18 & Tue 16-19	

GOOD LUCK!

Question 1. The resting pulse rates of 18 individuals are given in the following stem-and-leaf display.

Stem	Leaf
5	7
6	0 5
7	2 2 2 4 4 4 9
8	0 0 3 4 6 7
9	
10	2 8

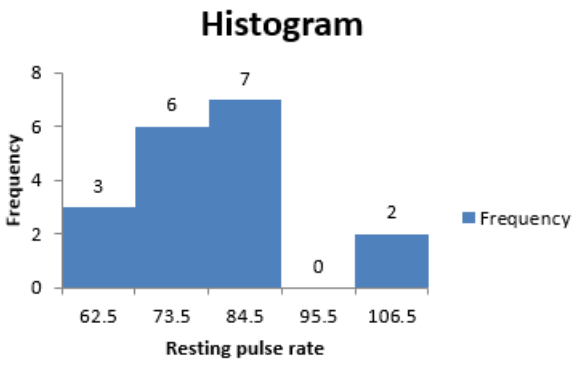
a) Using **five** classes, construct a frequency distribution table.
b) Draw the histogram for the given data.
c) Draw the ogive for the given data.

Solution:

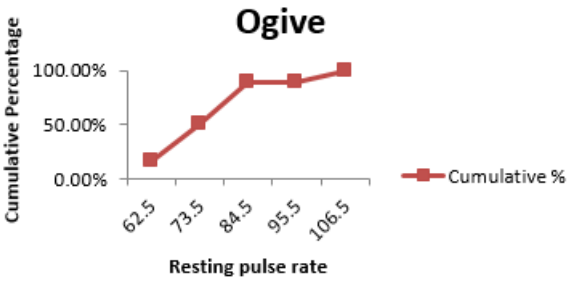
a) Sorted data: $57, 60, 65, 72, 72, 72, 74, 74, 74, 79, 80, 80, 83, 84, 86, 87, 102, 108$
$n = 18$, $k = 5 \Rightarrow$ class width$= \frac{\text{range}}{k} = \frac{108-57}{5} = \frac{51}{5} = 10.2$ (round up to 11!)

Classes	Frequencies	Percentages	Cumulative frequencies	Cumulative percentages
$[57, 68)$	3	16.67	3	16.67
$[68, 79)$	6	33.33	9	50.00
$[79, 90)$	7	38.89	16	88.89
$[90, 101)$	0	0.00	16	88.89
$[101, 112)$	2	11.11	18	100.00
Total	18	100.00		

b)



c)



Question 2. The speed trap (radar) on Mustafa Kemal Sahil Bulvari recorded the speed of cars of a specific brand as

86 74 80 72 87 60 108 72 80 74 83 102 79 74 84 72 57 65

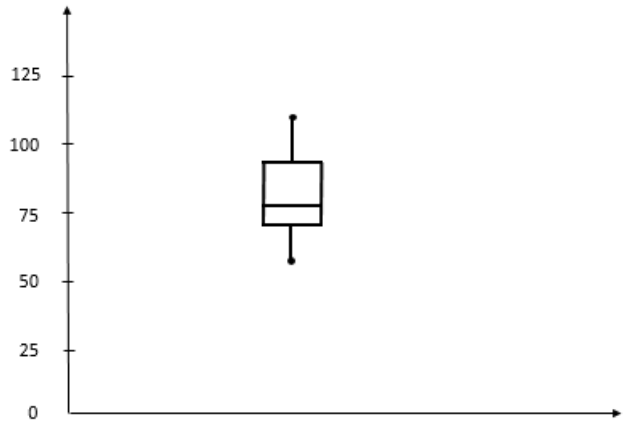
a) Find mean, median, and mode/s (if any) for the speed of these cars.
b) Find range and interquartile range (IQR) for the speed of these cars.
c) Draw the box-and-whisker plot for the speed of these cars.

Solution:
Sorted data: $57, 60, 65, 72, 72, 72, 74, 74, 74, 79, 80, 80, 83, 84, 86, 87, 102, 108$
$n = 18$

a) $\bar{x} = \frac{\sum_{i=1}^{18} x_i}{18} = \frac{1409}{18} = 78.28$
$\tilde{x}$ = value at $0.5(n+1) = 0.5(19) = 9.5$th ordered position $= 74 + 0.5(79 - 74) = 76.5$
There are two modes: 72 and 74.

b) range=max-min= $108 - 57 = 51$
Q_1 = value at $0.25(19) = 4.75$th ordered position $= 72 + 0.75(72 - 72) = 72$
Q_3 = value at $0.75(19) = 14.25$th ordered position $= 84 + 0.25(86 - 84) = 84.5$
IQR=$Q_3 - Q_1 = 84.5 - 72 = 12.5$

c)



Question 3. A random sample for five exam scores produced the following data values:

Hours Studied (x)	Exam Score (y)
3.5	88
2.4	76
4	92
5	85
1.1	60

a) Compute the covariance and the correlation coefficient.
b) Discuss the relationship between x and y by comparing $|r|$ with $\frac{2}{\sqrt{n}}$.

Solution:
$n = 5$
a)

x_i	y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
3.5	88	0.3	7.8	0.09	60.84	2.34
2.4	76	-0.8	-4.2	0.64	17.64	3.36
4	92	0.8	11.8	0.64	139.24	9.44
5	85	1.8	4.8	3.24	23.04	8.64
1.1	60	-2.1	-20.2	4.41	408.04	42.42
16	401	0	0	9.02	648.8	66.2

$\bar{x} = \frac{16}{5} = 3.2 \quad \bar{y} = \frac{401}{5} = 80.2$
$s_x^2 = \frac{9.02}{5-1} = 2.255 \Rightarrow s_x = \sqrt{2.255} = 1.50$
$s_y^2 = \frac{648.8}{5-1} = 162.2 \Rightarrow s_y = \sqrt{162.2} = 12.74$
$s_{xy} = \frac{66.2}{5-1} = 16.55$
$r = \frac{s_{xy}}{s_x s_y} = \frac{16.55}{(1.5)(12.74)} = 0.866$

b) $s_{xy} = 16.55 \Rightarrow x$ and y act together (as hours studied increases so do the exam scores).
$r = 0.866 > 0 \Rightarrow$ there is a positive linear relationship between x and y.
Since $|r| = 0.866 < 0.894 = \frac{2}{\sqrt{n}}$, the positive linear relationship is not strong.

Question 4. A regional sales manager of a computer company believes that their customers can be classified as younger, middle age, and older, and finds that of all customers 50%, 40% and 10%, respectively, fall into these categories. The manager found that a particular product was purchased by 60% of the younger customers, by 40% of middle age customers, and by 30% of older customers.

a) What is the probability that a randomly chosen customer purchases this product?

b) If this product is purchased, what is the probability that it was purchased by a younger customer?

Solution:

Defining the events

Y-"the customer is younger",

M-"the customer is of middle age",

O-"the customer is older", and

P-"the customer makes a purchase",

we are given that $P(Y) = 0.5$, $P(M) = 0.4$, $P(O) = 0.1$, $P(P|Y) = 0.6$, $P(P|M) = 0.4$, and $P(P|O) = 0.3$.

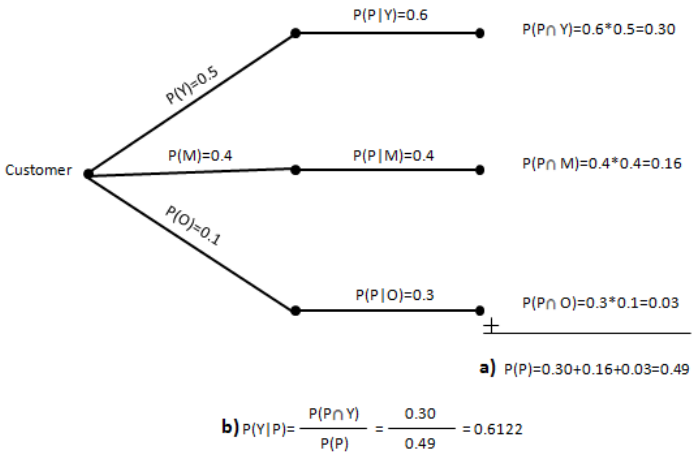
a) $P(P) = ?$

$$\begin{aligned} P(P) &= P(P \cap Y) + P(P \cap M) + P(P \cap O) \\ &= P(P|Y)\,P(Y) + P(P|M)\,P(M) + P(P|O)\,P(O) \\ &= (0.6)(0.5) + (0.4)(0.4) + (0.3)(0.1) \\ &= 0.30 + 0.16 + 0.03 = 0.49 \end{aligned}$$

b) $P(Y|P) = ?$

$$\begin{aligned} P(Y|P) &= \frac{P(P|Y)\,P(Y)}{P(P|Y)\,P(Y) + P(P|M)\,P(M) + P(P|O)\,P(O)} \\ &= \frac{(0.6)(0.5)}{(0.6)(0.5) + (0.4)(0.4) + (0.3)(0.1)} \\ &= \frac{0.30}{0.49} = 0.6122 \end{aligned}$$

OR



Question 5. A factory manager is considering whether to replace a temperamental machine. A review of past records indicates the following probability distribution for the number of breakdowns of this machine in a week.

Number of breakdowns	0	1	2	3	4
Probability	0.10	0.26	0.42	...	0.06

a) Find the probability that the number of breakdowns is 3 in a week.
b) Find the probability that the number of breakdowns in a week is less than 3.
c) Find mean and variance of the number of breakdowns in a week .
d) If the cost per breakdown in a week is estimated to be \$500, find the mean and variance of the breakdown cost per week.

Solution:

Let X denote the number of breakdowns for the machine per week.

a) $P\{X=3\} = P(3) = 1-[P(0)+P(1)+P(2)+P(4)] = 1-[0.10+0.26+0.42+0.06] = 1-0.84 = 0.16$

b) $P\{X<3\} = P(0)+P(1)+P(2) = 0.10+0.26+0.42 = 0.78$

c)

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	0.10	0	0	0
1	0.26	0.26	1	0.26
2	0.42	0.84	4	1.68
3	0.16	0.48	9	1.44
4	0.06	0.24	16	0.96
	1.00	1.82		4.34

$\mu_X = E(X) = 1.82$ and $E(X^2) = 4.34$.
$\sigma_X^2 = Var(X) = E(X^2) - \mu_X^2 = 4.34 - (1.82)^2 = 1.0276$.

d) If the cost per breakdown in a week is estimated to be \$500, then $C = 500X$.
$\mu_C = 500\mu_X = 500(1.82) = \$910$ and $\sigma_C^2 = (500)^2\sigma_X^2 = 250000(1.0276) = \$256900$.