Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Chapter 3: Elements of Chance: Probability Methods

Department of Mathematics
Izmir University of Economics

Week 3-4
2014-2015

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Introduction

In this chapter we will focus on

- the definitions of *random experiment*, *outcome*, and *event*,
- *probability* and its rules, and
- an important result that has many applications to management decision making: *Bayes' Theorem*.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Random Experiment, Outcomes, and Events

Definition:

A *random experiment* is a process leading to two or more possible outcomes, without knowing exactly which outcome will occur.

Some examples of random experiments:

- A coin is tossed and the outcome is either a head (H) or a tail (T).
- A company has the possibility of receiving $0 - 5$ contract awards.
- A die is rolled and the outcome is one of the six sides of the die.
- A customer enters a store and either purchases a shirt or does not.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Random Experiment, Outcomes, and Events

Definition:

The possible outcomes of a random experiment are called the *basic outcomes* and the set of all basic outcomes is called the *sample space* and is denoted by *S*.

In the experiment of

- tossing a coin the basic outcomes are head (H) and tail (T), so $S = \{H, T\}$,
- rolling a die the basic outcomes are $1, 2, 3, 4, 5$, and 6, so $S = \{1, 2, 3, 4, 5, 6\}$.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Random Experiment, Outcomes, and Events

Definition:

An *event*, E, is any subset of the sample space consisting of basic outcomes. The *null (impossible) event* represents the absence of a basic outcome and is denoted by $\emptyset$.

An event is said to occur if the random experiment results in one of the basic outcomes in that event.

In the experiment of rolling a die the basic outcomes are $1, 2, 3, 4, 5$, and 6, so $S = \{1, 2, 3, 4, 5, 6\}$.

Let *A* be the event that the number on the die is even and *B* be the event that the number on the die is odd.

Then $A = \{2, 4, 6\} \subset S$ and $B = \{1, 3, 5\} \subset S$.

Event *A* is said to be occur if and only if the result of the experiment is one of 2, 4, or 6. Similarly, event *B* occurs if and only if the result of the experiment is 1, 3, or 5.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Intersection of Events, Union of Events, and Complement of an Event

Let *A* and *B* be two events in the sample space *S*.

- The intersection of *A* and *B*, denoted by $A \cap B$, is the set of all basic outcomes in *S* that belong to both *A* and *B*.

- The union of *A* and *B*, denoted by $A \cup B$, is the set of all basic outcomes in *S* that belong to at least one of *A* and *B*.

- The complement of *A*, denoted by $\bar{A}$ or A', is the set of all basic outcomes in *S* that doesn't belong to *A*.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Mutually Exclusive Events

Definition:

If the events *A* and *B* have no common basic outcomes, they are called *mutually exclusive* and $A \cap B = \emptyset$.

Note: For any event *A*, *A* and $\bar{A}$ are always mutually exclusive.

Definition:

Given the *K* events $E_1, E_2, \ldots, E_K$ in the sample space *S*, if $E_1 \cup E_2 \cup \cdots \cup E_K = S$, these *K* events are said to be *collectively exhaustive*.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. A die is rolled. If events *A*, *B*, and *C* are defined as:
A :"result is even",
B :"result is at least 4", and
C :"result is less than 6",
describe the events

a) $A \cap B$

b) $A \cup B \cup C$

c) A'

d) $A \cup B$

e) $A \cap B \cap C$

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Probability

Suppose that a random experiment is to be carried out and we want to determine the probability that a particular event will occur.

Note: Probability is measured over the range from 0 to 1.

There are three definitions of probability:

1. Classical probability
2. Relative frequency probability
3. Subjective probability

In this course, we will focus only on the first one.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Classical Probability

Definition:

Classical probability is the proportion of times that an event will occur (assuming that all outcomes in a sample space are equally likely to occur). The probability of an event *A* is

$$P(A) = \frac{n(A)}{n(S)},$$

where $n(A)$ is the number of outcomes that satisfy the condition of event *A* and $n(S)$ is the total number of outcomes in the sample space.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. If two dice are rolled, what is the probability that the sum of the upturned faces will equal 7?

Solution.

$$
\begin{aligned}
S = \ & \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\
& (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\
& (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\
& (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\
& (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\
& (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\} \qquad \Rightarrow n(S) = 36
\end{aligned}
$$

$$
\begin{aligned}
A = \ & \text{sum of the upturned faces is 7} \\
= \ & \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\} \qquad \Rightarrow n(A) = 6
\end{aligned}
$$

Therefore, $P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6} = 0.1667$.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Generalized Basic Principle of Counting

If r experiments that are to be performed are such that the first one may result in any of n_1 possible outcomes, and for each of these n_1 possible outcomes there are n_2 possible outcomes of the second experiment, and if for each of the possible outcomes of the first two experiments there are n_3 possible outcomes of the third experiment, and if ..., then there is a total of $n_1 n_2 \cdots n_r$ possible outcomes of the r experiments.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Example. In a medical study patients are classified according to their blood types (A, B, AB, and 0) and according to their blood pressure levels (low, normal, and high). In how many different ways a patient can be classified?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Example. How many different outcomes are possible if a coin is tossed twice and a die is rolled?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Permutations and Combinations

1. Number of orderings

We begin with the problem of ordering. Suppose that we have *x* objects that are to be **placed in order** in such a way that each object may be used **only once**.
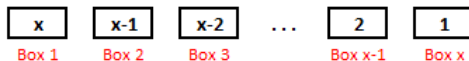
The total number of possible ways of arranging *x* objects **in order** is given by

$$x(x-1)(x-2)\cdots(2)(1) = x!,$$

where $x!$ is read "*x* factorial".

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Permutations and Combinations

2. Permutations

Suppose that now we have *n* objects with which the *x* **ordered boxes** could be filled ($n > x$) in such a way that each object may be used **only once**.

n	n-1	n-2	$\cdots$	n-x+2	n-x+1
Box 1	Box 2	Box 3		Box x-1	Box x
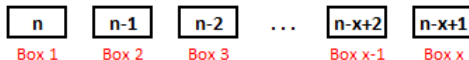
The total number of permutations of *x* object chosen from *n*, P_x^n, is the number of possible arrangements when *x* objects are to be **selected** from a total of *n* and **arranged in order**. This number is

$$P_x^n = n(n-1)(n-2)\cdots(n-x+2)(n-x+1) = \frac{n!}{(n-x)!}.$$

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Permutations and Combinations

3. Combinations

Suppose that we are interested in the number of different ways that x can be **selected** from n (where no object may be chosen more than once) but the **order is not important**.

The number of combinations of x object chosen from n, C_x^n, is the number of possible selections that can be made. This number is

$$C_x^n = \frac{P_x^n}{x!} = \frac{\frac{n!}{(n-x)!}}{x!} = \frac{n!}{(n-x)!x!} = \binom{n}{x}.$$

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Example. In how many different ways can a person invite three of her eight closest friends to a party?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Example. In how many ways can we choose three letters from A,B,C,D

a) if the order is important?

b) if the order is not important?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Example. In how many different ways can the letters in UNUSUALLY be arranged?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Example. In how many different ways can we arrange 10 books if 6 of them are on Mathematics, 3 are on Chemistry, 1 is on Physics?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Example. Consider a shuffled deck of 52 cards.

a) In how many different ways can an Ace (**A**) be drawn?

b) What is the probability of drawing an Ace (**A**)?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Example. There are five laptops of which three are brand *A* and two are brand *B*. Two of them will be chosen at random.

a) Find the sample space.

b) Define the event *E* by "One brand *A* and one brand *B* laptops will be chosen." and list the elements of event *E*.

c) What is the probability that one brand *A* and one brand *B* laptops will be chosen? (Find *P* (*E*).)

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Example. Suppose that there are ten brand *A*, five brand *B*, and four brand *C* laptops and three of them will be chosen at random. What is the probability that two of them will be brand *A* and one will be brand *C*?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Counting, Permutations, and Combinations

Example. A manager is available a pool of 8 employees who could be assigned to a project-monitoring task. 4 of the employees are women and 4 are men. 2 of the men are brothers. The manager is to make the assignment at random so that each of the 8 employees is equally likely to be chosen. Let *A* be the event that "Chosen employee is a man." and *B* be the event that "Chosen employee is one of the brothers."

a) Find $P(A)$.

b) Find $P(B)$.

c) Find $P(A \cap B)$.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Probability Rules

Rules of probability are:

1. $P(S) = 1$

2. For any event $A \subset S$, $0 \leq P(A) \leq 1$

3. If $A_1, A_2, \ldots$ is a countable collection of mutually exclusive events, then
   $P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$

4. For any event $A \subset S$, $P(\bar{A}) = 1 - P(A)$

5. $P(\emptyset) = 0$

6. For any two events $A \subset S$ and $B \subset S$,
   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. The probability of *A* is 0.60, the probability of *B* is 0.45, and the probability of both is 0.30.

a) What is the probability of either *A* and *B*?

b) What are $P(\bar{A})$ and $P(\bar{B})$?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. A pair of dice is rolled. If *A* is the event that a total of 7 is rolled and *B* is the event that at least one die shows up 4, find the probabilities for *A*, *B*, *A* ∩ *B*, and *A* ∪ *B*.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. A corporation has just received new machinery that must be installed and checked before it becomes operational. The accompanying table shows a manager's probability assessment for the number of days required before the machinery becomes operational.

Number of days	3	4	5	6	7
Probability	0.08	0.24	0.41	0.20	0.07

Let *A* be the event "It will be more than four days before the machinery becomes operational." and let *B* be the event "It will be less than six days before the machinery becomes available."

a) $P(A) = ?$

b) $P(B) = ?$

c) $P(\bar{A}) = ?$

d) $P(A \cap B) = ?$

e) $P(A \cup B) = ?$

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Conditional Probability

Suppose that two fair dice were rolled and we saw that one of them is 3. Under this condition, what is the probability of getting a total of 5?

Since we know that one of the the dice is 3, we will be dealing with the restricted sample space

$S_R = \{(1,3),(2,3),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$

instead of the whole sample space $S = \{(1,1),(1,2),\ldots(6,6)\}$ consisting of 36 basic outcomes.

Therefore the desired event is $\{(2,3),(3,2)\}$ and the desired probability is $\frac{2}{11} = 0.1818$.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Conditional Probability

Definition:

Let *A* and *B* be two events. The *conditional probability* of event *A*, given that event *B* has occurred, is denoted by $P(A|B)$ and is defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ provided that } P(B) > 0.$$

Similarly,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \text{ provided that } P(A) > 0.$$

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. An international cargo company knows that 75% of its customers prefer shipments with SMS support while 80% prefer shipments with internet support. It is also known that 65% of the customers prefer both. What are the probabilities that

a) a customer who prefer SMS support will also prefer internet support?

b) a customer who prefer internet support will also prefer SMS support?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. The probability that there will be a shortage of cement is 0.28 and the probability that there will not be a shortage of cement and a construction job will be finished on time is 0.64. What is the probability that the construction job will be finished on time given that there will not be a shortage of cement?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Multiplication Rule

Definition:

Let A and B be two events. Using the definitions of conditional probabilities $P(A|B)$ and $P(B|A)$, we have

$$P(A \cap B) = P(A|B)P(B)$$

$$P(A \cap B) = P(B|A)P(A).$$

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. What is the probability of getting 2 Aces (**A**) when two cards are drawn randomly from an ordinary deck of 52 cards?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Statistical Independence

Definition:

Let A and B be two events. A and B are said to be *statistically independent* if and only if

$$P(A \cap B) = P(A) P(B).$$

More generally, the events $E_1, E_2, \ldots, E_K$ are mutually statistically independent if and only if

$$P(E_1 \cap E_2 \cap \cdots \cap E_K) = P(E_1) P(E_2) \cdots P(E_K).$$

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Statistical Independence

If A and B are (statistically) independent, then

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\,P(B)}{P(B)} = P(A)\text{, provided that } P(B) > 0$$

and

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)\,P(B)}{P(A)} = P(B)\text{, provided that } P(A) > 0.$$

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Statistical Independence

Note: Do not confuse independent events with mutually exclusive events:

$$A \text{ and } B \text{ are independent} \iff P(A \cap B) = P(A)P(B)$$

$$A \text{ and } B \text{ are mutually exclusive} \iff A \cap B = \emptyset \iff P(A \cap B) = 0.$$

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. Consider drawing a card from an ordinary deck of 52 cards. Let
A :"Getting a queen (**Q**)" and *B* :"Getting a spade (♠). Are the events *A* and
B independent?"

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. Experience about a specific model of a mobile phone is that 80% of this model will operate for at least one year before repair is required. A director buys three of these phones. What is the probability that all three phones will work for at least one year without a problem?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. A quality-control manager found that 30% of worker-related problems occurred on Mondays and that 20% occurred in the last hour of a day's shift. It was also found that 4% of the worker-related problems occurred in the last hour of Monday's shift.

a) What is the probability that a worker-related problem that occurs on a Monday does not occur in the last hour of the day's shift?

b) Are the events "problem occurs on Monday" and "problem occurs in the last hour of a day's shift" statistically independent?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Bivariate Probabilities

Consider two distinct sets of events $A_1, A_2, \ldots, A_H$ and $B_1, B_2, \ldots, B_K$. The events A_i, $i = 1, 2, \ldots H$ and B_j, $j = 1, 2, \ldots K$ are mutually exclusive and collectively exhaustive within their sets but intersections $A_i \cap B_j$ can occur between all events from the two sets.

The following table illustrates the outcomes of bivariate events:

	B_1	B_2	.	.	.	B_K	TOTAL
A_1	$P(A_1 \cap B_1)$	$P(A_1 \cap B_2)$	.	.	.	$P(A_1 \cap B_K)$	$P(A_1)$
A_2	$P(A_2 \cap B_1)$	$P(A_2 \cap B_2)$	.	.	.	$P(A_2 \cap B_K)$	$P(A_2)$
.	.	.				.	.
.	.	.				.	.
.	.	.				.	.
A_H	$P(A_H \cap B_1)$	$P(A_H \cap B_2)$	.	.	.	$P(A_H \cap B_K)$	$P(A_H)$
TOTAL	$P(B_1)$	$P(B_2)$	.	.	.	$P(B_K)$	1

Note: If the probabilities of all intersections are known, then the whole probability structure of the random experiment is known and other probabilities of interest can be calculated.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Bivariate Probabilities

Definition:

The intersection probabilities $P(A_i \cap B_j)$, $i = 1, 2, \ldots H$ and B_j, $j = 1, 2, \ldots K$, are called *joint probabilities*. The probabilities for individual events $P(A_i)$, $i = 1, 2, \ldots H$, and $P(B_j)$, $j = 1, 2, \ldots K$, are called *marginal probabilities*.

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Bivariate Probabilities

Example. A potential advertiser wants to know both income and other relevant characteristics of the audience for a particular television show. Families may be categorized, using A_i, as to whether they regularly, occasionally, or never watch a particular series. In addition, they can be categorized, using B_j, according to low-, middle-, and high-income subgroups. Find all marginal probabilities.

Watching Income frequency	B_1: low-	B_2: middle-	B_3: high-	TOTAL
A_1: regular	0.04	0.13	0.04	
A_2: occasionally	0.06	0.11	0.1	
A_3: never	0.22	0.17	0.13	
TOTAL				

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Bivariate Probabilities

We can find any conditional probability $P(A_i|B_j)$ and $P(B_j|A_i)$ if we know all
the joint probabilities. Some conditional probabilities for the previous example
are:

$$P(A_1|B_1) = \frac{P(A_1 \cap B_1)}{P(B_1)} = \frac{0.04}{0.32} = 0.125$$

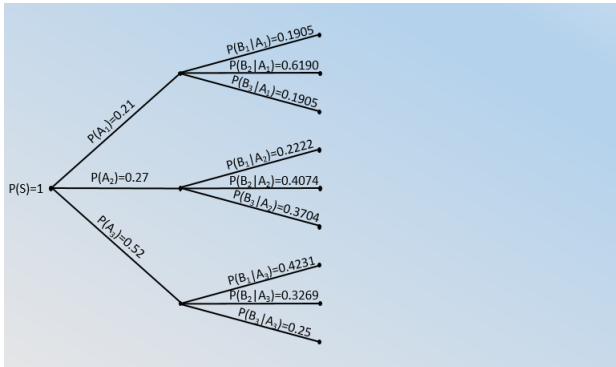$$P(A_2|B_1) = \frac{P(A_2 \cap B_1)}{P(B_1)} = \frac{0.06}{0.32} = 0.1875$$

$$P(A_3|B_1) = \frac{P(A_3 \cap B_1)}{P(B_1)} = \frac{0.22}{0.32} = 0.6875$$

$$P(B_2|A_1) = \frac{P(A_1 \cap B_2)}{P(A_1)} = \frac{0.13}{0.21} = 0.6190$$

$$P(B_3|A_3) = \frac{P(A_3 \cap B_3)}{P(A_3)} = \frac{0.13}{0.52} = 0.25$$

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
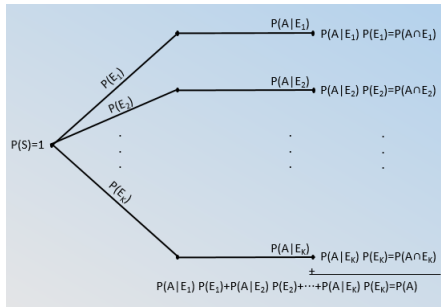Bayes' Theorem

Bivariate Probabilities

We can construct a tree diagram using marginal and conditional probabilities and then obtain the joint probabilities using multiplication rule. The tree diagram for the previous example is:

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Bayes' Theorem

If $E_1, E_2, \ldots, E_K$ are mutually exclusive and collectively exhaustive events, then for any event A with $P(A) \neq 0$ and for any $i = 1, 2, \ldots K$

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + \cdots + P(A|E_K)P(E_K)}$$

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Bayes' Theorem

Example. A hotel rents cars for its guests from three rental agencies. It is known that 25% are from agency X, 25% are from agency Y, and 50% are from agency Z. If 8% of the cars from agency X, 6% from agency Y, and 15% from agency Z need tune-ups, what is the probability that a car needing a tune-up come from agency Y?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Bayes' Theorem

Example. A life insurance salesman finds that, of all the sales he makes, 70% are to people who already own policies. He also finds that, of all contacts for which no sale is made, 50% already own life insurance policies. Furthermore, 40% of all contacts result in sales. What is the probability that a sale will be made to a contact who already owns a policy?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Bayes' Theorem

Example. In a large city, 8% of the inhabitants have contracted a particular disease. A test for this disease is positive in 80% of people who have the disease and is negative in 80% of people who do not have the disease. What is the probability that a person for whom the test result is positive has the disease?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. A record-store owner assesses customers entering the store as high school age, college age, or older, and finds that of all customers 30%, 50%, and 20%, respectively, fall into these categories. The owner also found that purchases were made by 20% of high school age customers, by 60% of college age customers, and by 80% of older customers.

a) What is the probability that a randomly chosen customer entering the store will make a purchase?

b) If a randomly chosen customer makes a purchase, what is the probability that this customer is high school age?

Introduction
Random Experiment, Outcomes, and Events
Probability
Probability Rules
Bivariate Probabilities
Bayes' Theorem

Example. A restaurant manager classifies customers as regular, occasional, or new, and finds that of all customers 50%, 40%, and 10%, respectively, fall into these categories. The manager found that wine was ordered by 70% of the regular customers, by 50% of the occasional customers, and by 30% of the new customers.

a) What is the probability that a randomly chosen customer orders wine?

b) If wine is ordered, what is the probability that the person ordering is a regular customer?

c) If wine is ordered, what is the probability that the person ordering is an occasional customer?