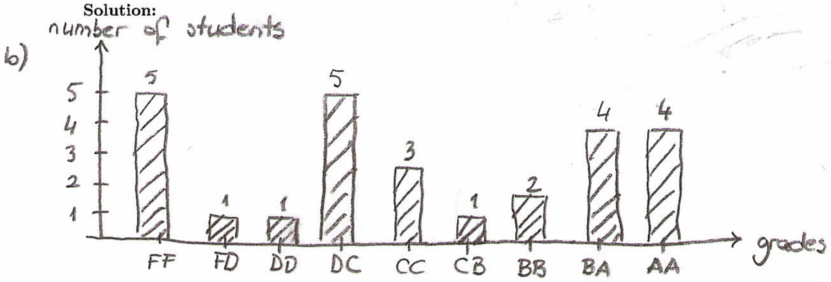
Question 1. The grades obtained by the students of one course are given in the following frequency distribution table:

CLASSES Grades (Letter Grades)	FREQUENCIES Number of Students	CUMULATIVE FREQUENCIES a)
0.00 (FF)	5	5
0.50 (FD)	1	6
1.00 (DD)	1	7
1.50 (DC)	5	12
2.00 (CC)	3	15
2.50 (CB)	1	16
3.00 (BB)	2	18
3.50 (BA)	4	22
4.00 (AA)	4	26

a) Fill the Cumulative Frequencies column.
b) Graph the bar chart of the given data.
c) Find the mean, median, and mode/s (if any) of the given data.
d) Find the range and interquartile range (IQR) of the given data.

Solution:

b)



c) $\bar{x} = \dfrac{5(0)+1(0.5)+1(1)+5(1.5)+3(2)+1(2.5)+2(3)+4(3.5)+4(4)}{26}$

$= \dfrac{53.5}{26} \cong 2.06$ //

$\tilde{x}$ = value at $0.5(27) = 13.5^{th}$ ordered position $= 2.00$ //

modes = 0.00 and 1.50 //

d) range = max − min = 4.00 − 0.00 = 4.00 //

Q_1 = value at $0.25(27) = 6.75^{th}$ ordered position

$= 0.50 + (0.75)(1.00 - 0.50) = 0.875$

Q_3 = value at $0.75(27) = 20.25^{th}$ ordered position

$= 3.50 + (0.25)(3.50 - 3.50) = 3.50$

$IQR = Q_3 - Q_1 = 3.50 - 0.875 = 2.625$ //

Question 2. A professor finds that she awards a final grade of A to 20% of her students. Of those who obtain a final grade of A, 70% obtained an A on the midterm examination. Also, 10% of the students who failed to obtain a final grade of A earned an A on the midterm exam. What is the probability that a student with an A on the midterm examination will obtain a final grade of A?

Solution:

Let

F - "student obtains a final grade of A"

M - "student obtains an A on the midterm exam"

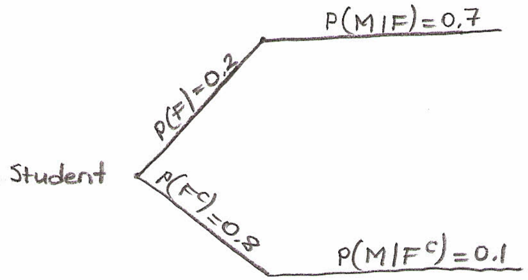
$P(F) = 0.20$ $\quad P(M|F) = 0.70$ $\quad P(M|F^c) = 0.10$ $\quad P(F|M) = ?$

Using Bayes' theorem,

$$P(F|M) = \frac{P(M|F)P(F)}{P(M|F)P(F) + P(M|F^c)P(F^c)}$$

$$= \frac{(0.7)(0.2)}{(0.7)(0.2) + (0.1)(0.8)} \cong 0.6364 \text{ //}$$

OR



$P(M \cap F) = 0.14$

$+ \; P(M \cap F^c) = 0.08$

$P(M) = 0.22$

$$P(F|M) = \frac{P(F \cap M)}{P(M)} = \frac{0.14}{0.22} \cong 0.6364 \text{ //}$$

Question 3. A company specializes in installing and servicing central-heating furnaces. In this prewinter period, service calls may result in an order for a new furnace. The following table shows estimated probabilities for the numbers of new furnace orders generated in this way in the last 2 weeks of September.

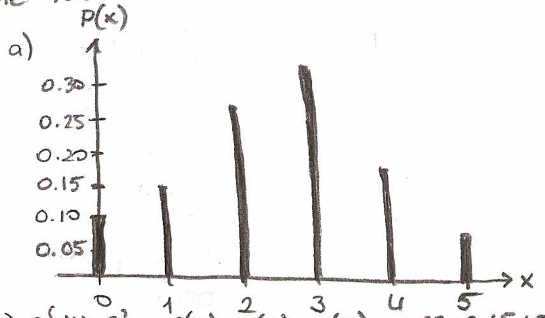
Number of Orders	0	1	2	3	4	5
Probability	0.10	0.14	0.26	0.28	...	0.07

a) Graph the probability distribution.
b) Calculate the cumulative probability distribution.
c) Find the probability that at least 3 orders will be generated in this period.
d) Find the mean and the standard deviation of the number of orders for new furnaces in this period.
e) If the profit per order in the last 2 weeks of September of this company is estimated to be $250, find the mean and standard deviation of the profit for this period.

Solution:

Let X be the number of orders of the new furnace in the last 2 weeks of September.

a)



b)

x	P(x)	F(x)
0	0.10	0.10
1	0.14	0.24
2	0.26	0.50
3	0.28	0.78
4	0.15	0.93
5	0.07	1.00
	1.00	

c) $P\{X \geq 3\} = P(3) + P(4) + P(5) = 0.28 + 0.15 + 0.07 = 0.50$

d)

x	P(x)	x P(x)	x^2	$x^2 P(x)$
0	0.10	0	0	0
1	0.14	0.14	1	0.14
2	0.26	0.52	4	1.04
3	0.28	0.84	9	2.52
4	0.15	0.60	16	2.40
5	0.07	0.35	25	1.75
	1.00	2.45		7.85

$$\mu_X = \sum_{x=0}^{5} x P(x) = 2.45$$

$$E(X^2) = \sum_{x=0}^{5} x^2 P(x) = 7.85$$

$$\sigma_X^2 = E(X^2) - \mu_X^2 = 1.8475$$

$$\sigma_X = \sqrt{\sigma_X^2} \cong 1.3592$$

e) If P denotes the profit per order in the last 2 weeks of September of this company, then $P = 250X$.

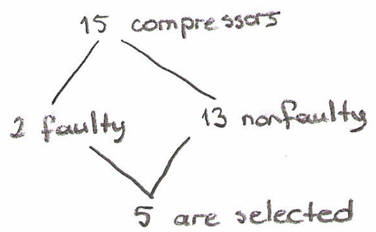
$$\mu_P = 250\mu_X = 250(2.45) = \$\,612.5$$

$$\sigma_P = |250|\,\sigma_X = 250(1.3592) \cong \$\,339.8$$

Question 4. A large company has an inspection system for the batches of small compressors purchased from vendors. A batch typically contains 15 compressors. In the inspection system, a random sample of 5 is selected and all are tested. Suppose there are 2 faulty compressors in the batch of 15.

a) What is the probability that for a given sample there will be 1 faulty compressor?
b) What is the probability that inspection will discover both faulty compressors?
c) What is the expected number of the faulty compressors under inspection?

Solution:



If X is the number of faulty compessors in the inspection system, then X is a hypergeometric random variable with $N = 15$, $s = 2$, and $n = 5$.

$$P(x) = P\{X = x\} = \frac{\binom{2}{x}\binom{13}{5-x}}{\binom{15}{5}}, \quad \max(0, 5-15+2) \leq x \leq \min(2,5)$$
$$0 \leq x \leq 2$$

a) $P\{X=1\} = P(1) = \dfrac{\binom{2}{1}\binom{13}{4}}{\binom{15}{5}} = \dfrac{2\,\frac{13\cdot12\cdot11\cdot10}{4\cdot3\cdot2\cdot1}}{\frac{15\cdot14\cdot13\cdot12\cdot11}{5\cdot4\cdot3\cdot2\cdot1}} = \dfrac{20}{42} \cong 0.4762$

b) $P\{X=2\} = P(2) = \dfrac{\binom{2}{2}\binom{13}{3}}{\binom{15}{5}} = \dfrac{1\cdot\frac{13\cdot12\cdot11}{3\cdot2\cdot1}}{\frac{15\cdot14\cdot13\cdot12\cdot11}{5\cdot4\cdot3\cdot2\cdot1}} = \dfrac{2}{21} \cong 0.0952$

c) $\mu_X = n\dfrac{s}{N} = 5\,\dfrac{2}{15} \cong 0.6667$

OR

x	P(x)	x P(x)
0	0.4286	0
1	0.4762	0.4762
2	0.0952	0.1904
		0.6666

$$\mu_X = \sum_{x=0}^{2} x P(x) = 0.6666$$