IUE – MATH 212 – Intro. to Prob. and Stat. II

First Midterm Exam — March 24, 2015 — 18:30 - 20:00

Name Surname : ____________________ ID # : ______________

KEY

Q1	Q2	Q3	Q4	TOTAL
25	25	25	25	100

- The exam consists of 4 questions of equal weight.
- Please read the questions carefully.
- Write your answers in the empty space at the end of each question. Be neat.
- Show all your work. Correct answers without sufficient explanation might not get full credit.
- Exchange of any material (e.g., calculators, rubbers, tables, etc.) is not allowed.
- Dictionaries and mobile phones are not allowed.
- You may use any empty space for scratch work.

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GOOD LUCK!

Question 1. The checking account balance at a local bank is normally distributed with a mean of $\mu = 1742$ \$ and a standard deviation of $\sigma = 132$\$. Suppose you select a random sample of 30 accounts.

a) What is the probability that the mean balance for these 30 accounts is less than 1,700\$?
b) The probability is 0.95 that the sample mean is less than what amount?

Solution:

a)

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < \frac{1700 - 1742}{132/\sqrt{30}} = -1.74$$ Using normal distribution.

The related probability would be

$$1 - P(z \leq 1.74) = 1 - 0.9591 = 0.0409$$

b) We need to determine the amount k such that

$$P(\bar{x} < k) = 0.95$$

Thus $P^{-1}(0.95) = 1.64$ and

$$\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < \frac{k - 1742}{132/\sqrt{30}} = 1.64 \Rightarrow k = 1781.52$$

Question 2. Candidates for employment at Health department of IEU, are required to take a test. Scores on this test are normally distributed with mean $\mu = 260$ and standard deviation $\sigma = 51$. A random sample of nine test scores was taken. What is the probability that the sample standard deviation is greater than 66?

Solution:

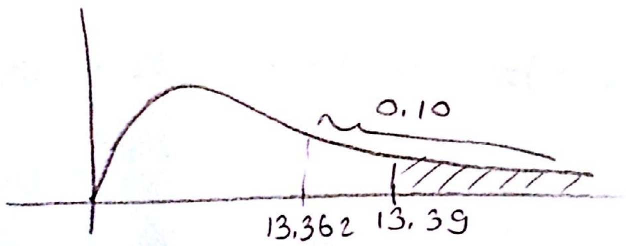
We need to determine that $P(s > 66)$.

Then

$$\chi^2_{n-1} = \frac{(n-1)s^2}{\sigma^2} > \frac{8 \times (66)^2}{(51)^2} = 13.39$$

and $\chi^2_{9,\,0.10} = 13.362$

Since $13.362 < 13.39$, the probability that $s < 66$ is less 0.10. i.e



Question 3. The manager of a local fitness center wants an estimate of the number of times members use the weight room per month. The monthly number of visits follows a normal distribution with standard deviation of 11 visits. From a random sample of 16 members, the average number of visits was 63 visits.

a) Find a 95% confidence interval for the population mean score of the applicants.
b) If a 70% confidence level is given, find $z_{\frac{\alpha}{2}}$.

Solution:

$\sigma = 11, \; n = 16, \; \bar{x} = 63$

a) $\bar{x} - z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}$

$63 - 1.96 \times \frac{11}{4} < \mu < 63 + 1.96 \times \frac{11}{4}$

$C.I = (57.61, \; 58.39)$

b) Since $70 = 100(1-\alpha)$ we have $\alpha = 0.3$ and $\frac{\alpha}{2} = 0.15$

Thus using standard normal distrib.

$P^{-1}(1 - 0.15) = P^{-1}(0.85) = 1.04$

i.e $z_{\frac{\alpha}{2}} = 1.04$

Question 4. From a random sample of 400 registered voters in a city, 320 indicated that they would vote in favor of a proposed policy in an upcoming election. 30 would vote against this policy and 50 would have no opinion. Find a 95% confidence interval for the population proportion in favor of this policy.

Solution:

Given $\hat{p} = \frac{320}{400} = 0.8$, $z_{\frac{\alpha}{2}} = 1.96$

The desired confidence interval is

$$\hat{p} - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\Rightarrow \quad 0.8 - 1.96\sqrt{\frac{0.8 \times 0.2}{400}} < p < 0.8 + 1.96\sqrt{\frac{0.8 \times 0.2}{400}}$$

$$\Rightarrow \quad 0.8 - 0.039 < p < 0.8 + 0.039$$

C.I $= (0.7608, 0.839)$

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