KEY

25 points	25 points	25 points	25 points	100 points
1	2	3	4	**Total**

MATH 153 CALCULUS I 18.12.2015

İzmir University of Economics Faculty of Arts and Sciences, Department of Mathematics

Midterm Exam 2

Student Name and Surname: ....

(1) Evaluate the indicated limit. Apply L-R:

(a) $\lim_{x\to 0} \frac{\sqrt{1+x}-1-\frac{x}{2}}{x^2}$ $\left(\frac{0}{0}\right)$

$$= \lim_{x\to 0} \frac{\frac{1}{2\sqrt{1+x}} - \frac{1}{2}}{2x} \quad \left(\frac{0}{0}\right)$$

Apply L-R:

$$= \lim_{x\to 0} \frac{\frac{-1}{4\sqrt{(1+x)^3}}}{2}$$

$$= \frac{-1}{8}$$

(b) $\lim_{x\to \frac{\pi}{2}} \frac{\sec x}{1+\tan x}$ $\left(\frac{\infty}{\infty}\right)$

$$= \lim_{x\to \frac{\pi}{2}} \frac{\cancel{\sec} x \cdot \tan x}{\sec^2 x} = \lim_{x\to \frac{\pi}{2}} \frac{\tan x}{\sec x}$$

$$= \lim_{x\to \frac{\pi}{2}} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}}$$

$$= \lim_{x\to \frac{\pi}{2}} \sin x$$

$$= 1$$

(c) $\lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$

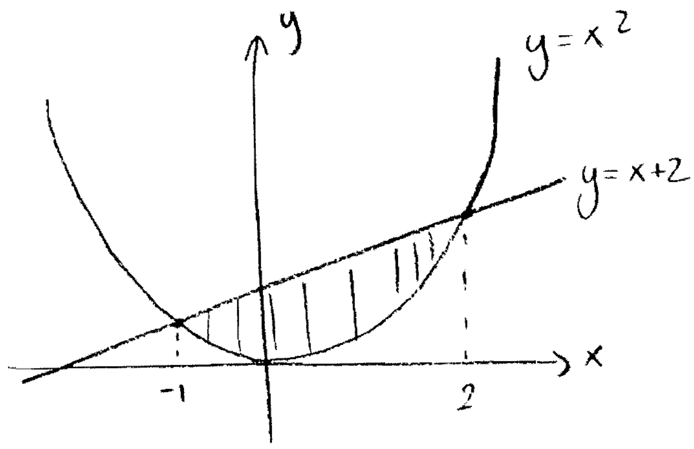
$$= \lim_{x\to 0} \frac{x-\sin x}{x\cdot \sin x} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{1-\cos x}{\sin x + x\cdot\cos x} \quad ; \cos 0 = 1$$

$$= \lim_{x\to 0} \frac{\sin x}{\cos x + \cos x - x\sin x}$$

$$= 0$$

(2) (a) Sketch and find the area of the region bounded by $y = x^2$ and the line $y = x + 2$.



$$\text{Area} = \int_{-1}^{2} (x+2-x^2)\,dx$$

$$= \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)\Big|_{-1}^{2}$$

$$= \left(2+4-\frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= \frac{9}{2}$$

Intersection points:

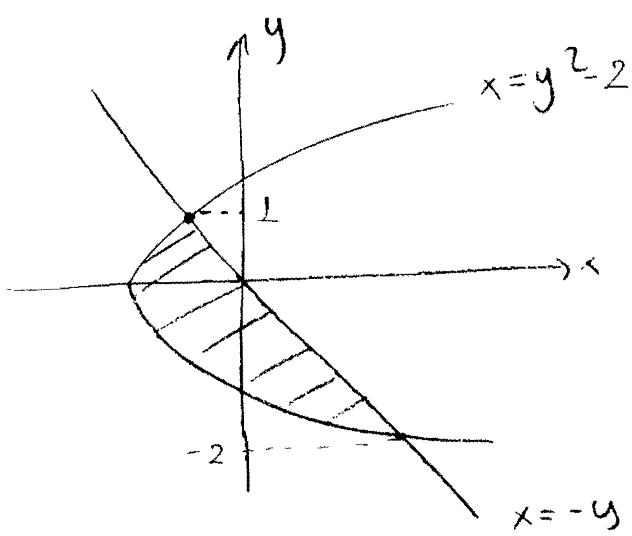
$x^2 = x+2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad , x = -1$

(b) Sketch and find the area of the region bounded by the graphs of the functions $x = y^2 - 2$ and $x = -y$.



$$\text{Area} = \int_{-2}^{1} (-y-y^2+2)\,dy$$

$$= \left(-\frac{y^2}{2} - \frac{y^3}{3} + 2y\right)\Big|_{-2}^{1}$$

$$= \left(-\frac{1}{2} - \frac{1}{3} + 2\right) - \left(-2 + \frac{8}{3} - 4\right)$$

$$= \frac{9}{2}$$

Intersection points:

$y^2 - 2 = -y$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = -2, \; y = 1$

(3) Let $f(x) = x^4 + 4x^3$.

(*a*) Find the domain and intercepts.

Domain: $\mathbb{R}$

x-intercept:

$f(x) = 0$

$x^4 + 4x^3 = x^3(x+4) = 0$

$x = 0 \ ; \ x = -4$

$(0,0) \quad (-4,0)$

y-intercept:

$x = 0 \Rightarrow f(0) = 0$

$(0,0)$

(*b*) Find the intervals of increase and decrease and the local extreme points.

$f'(x) = 4x^3 + 12x^2 = 0$

$4x^2(x+3) = 0$

$x = 0 \ , \ x = -3$

$f'(x)$: $-$ | -3 | $+$ | 0 | $+$

$f(x)$: dec. | -3 | inc. | 0 | inc.

local min

$f(-3) = -27$

(*c*) Find the intervals of concavity and the inflection points.

$f''(x) = 12x^2 + 24x = 0$

$12x(x+2) = 0$

$x = 0 \ , \ x = -2$

$f''(x)$: $+$ | -2 | $-$ | 0 | $+$

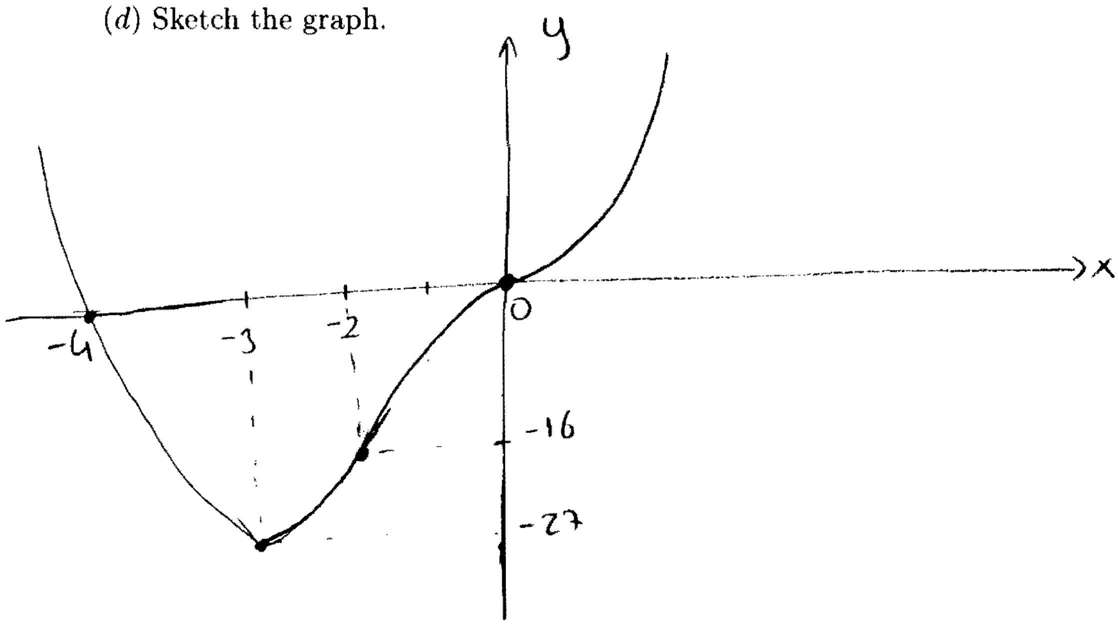
graph of $f(x)$: c-up | -2 | c-down | 0 | c-up

$f(-2) = -16$

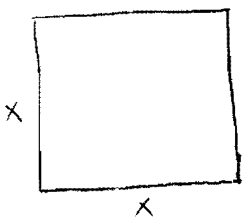
$f(0) = 0$

are inflection points

(*d*) Sketch the graph.



(4) (*a*) The area of a square is decreasing at a rate of $4.4 ft^2/min$. At the instant the area is $81 ft^2$, how fast is the side of the square changing?



Area : $A = 81\ ft^2$ $\qquad A = 81 = x^2 \Rightarrow x = 9\ ft$

$\frac{dA}{dt} = -4.4\ ft^2/min$

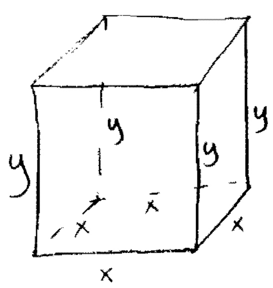
$\frac{dx}{dt} = ?$

$A = x^2$

$\frac{dA}{dt} = 2x \cdot \frac{dx}{dt}$

$-4.4 = 2 \cdot 9 \cdot \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -0.244\ ft/min$

(*b*) Of all boxes with a square base and a volume $100m^3$, which one has the minimum surface area? (Give its dimensions)



Area of base and top : $2x^2$

Area of four faces : $4xy$

Total area = $2x^2 + 4xy$

Volume = $100m^3$

Minimize $2x^2 + 4xy$ $\qquad$ If $x^2 y = 100$ then $y = \frac{100}{x^2}$

subject to $x^2 y = 100$

Let $f(x) = 2x^2 + 4x \cdot \frac{100}{x^2} = 2x^2 + \frac{400}{x}$. find min of f:

$f'(x) = 4x - \frac{400}{x^2} = \frac{4x^3 - 400}{x^2} = 0 \Rightarrow x = \sqrt[3]{100}$

$\Downarrow$

$y = \sqrt[3]{100}$