25 points	25 points	25 points	25 points	100 points
1	2	3	4	**Total**

MATH 153 CALCULUS I 08.01.2016

İzmir University of Economics Faculty of Arts and Sciences, Department of Mathematics

Final Exam

Student Name and Surname: ....

(1) Evaluate each integral using a suitable integration technique.

(a) $\int (3x+5)\cos x\,dx$

$= (3x+5)\cdot\sin x - \int \sin x \cdot 3dx$

$u = 3x+5 \qquad dv = \cos x \cdot dx$

$du = 3dx \qquad v = \sin x$

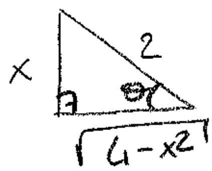
$\int u\cdot dv = u\cdot v - \int v\cdot du$

$= (3x+5)\cdot\sin x + 3\cos x + c$

(b) $\int \frac{1}{(4-x^2)^{3/2}}\,dx$

$= \int \frac{1}{8\cos^{\cancel{3}2}\theta}\cdot 2\cancel{\cos\theta}\,d\theta$

$\sin\theta = \frac{x}{2}$



$x = 2\sin\theta$

$dx = 2\cos\theta\cdot d\theta$

$(4-x^2)^{3/2} = (4-4\sin^2\theta)^{3/2}$

$= (4\cos^2\theta)^{3/2}$

$(4-x^2)^{3/2} = 8\cos^3\theta$

$= \frac{1}{4}\int \sec^2\theta\cdot d\theta$

$= \frac{1}{4}\tan\theta + c$

$= \frac{1}{4}\frac{x}{\sqrt{4-x^2}} + c$

(c) $\int \frac{18}{(x+3)(x^2-4)}\,dx$

$= \int \left(\frac{18/5}{x+3} + \frac{18/20}{x-2} - \frac{18/4}{x+2}\right)dx$

$\frac{18}{(x+3)(x-2)(x+2)} = \frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{x+2}$

$/x^2-4 \qquad /(x+3)(x+2) \qquad /(x+3)(x-2)$

$18 = A(x^2-4) + B(x+3)(x+2) + C(x+3)(x-2)$

$x=-3 \Rightarrow 18 = A\cdot 5 \Rightarrow A = 18/5$

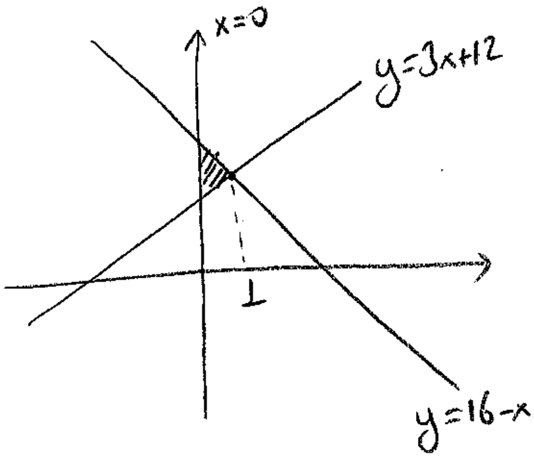
$x=2 \Rightarrow 18 = B\cdot 5\cdot 4 \Rightarrow B = 18/20$

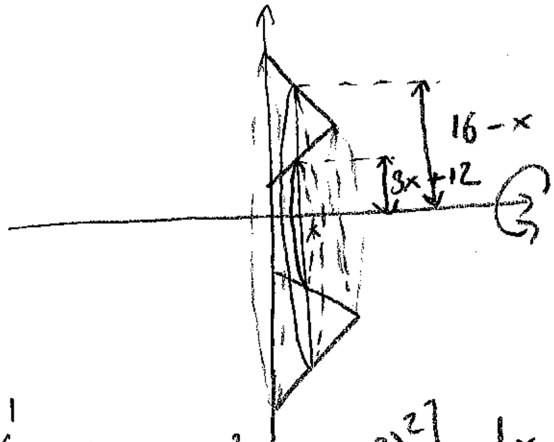
$x=-2 \Rightarrow 18 = C\cdot 1\cdot(-4) \Rightarrow C = -18/4$

$= \frac{18}{5}\ln|x+3| + \frac{18}{20}\ln|x-2| - \frac{18}{4}\ln|x+2| + C$

(2) Find the volume of the solid generated by rotating the triangular region bounded by $y = 16 - x$, $y = 3x + 12$, $x = 0$ about

(*a*) the x-axis using slicing method.





$3x+12=16-x$

$4x=4$

$x=1$

$$V=\int_0^1 \pi\left[(16-x)^2-(3x+12)^2\right]\cdot dx$$

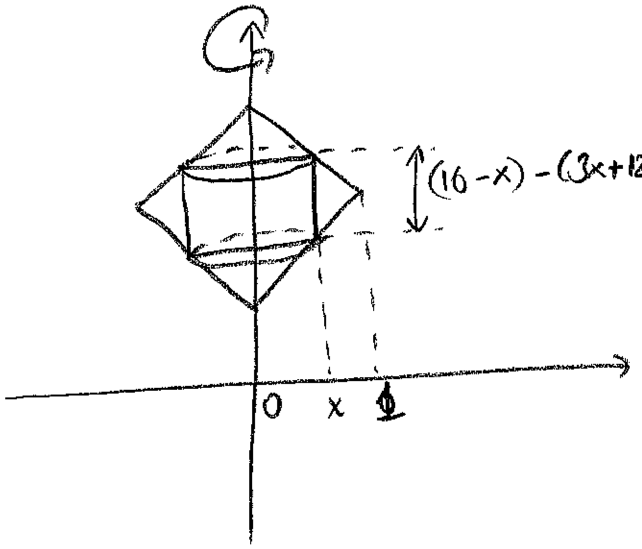
$$V=\pi\int_0^1 \left[256-32x+x^2-9x^2-72x-144\right]dx$$

$$V=\pi\int_0^1 (112-104x-8x^2)\,dx$$

$$=\pi\left(112x-104\frac{x^2}{2}-8\frac{x^3}{3}\right)\Big|_0^1$$

$$=\pi\cdot\frac{172}{3}$$

(*b*) the y-axis using cylindrical shell method.

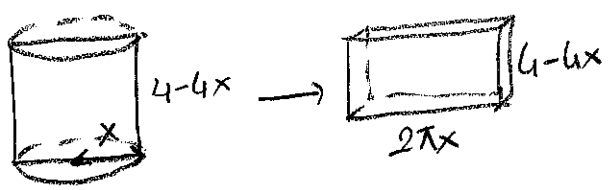


$$V=\int_0^1 2\pi x\,(4-4x)\,dx$$

$$=2\pi\int_0^1 (4x-4x^2)\,dx$$

$$=8\pi\left(\frac{x^2}{2}-\frac{x^3}{3}\right)\Big|_0^1$$

$$=\frac{8\pi}{6}$$



(3) Evaluate each <u>improper</u> integral or show that it diverges.

(a)
$$\int_0^{\pi/2} \frac{\sin x}{\cos^2 x} \cdot dx$$

$u = \cos x$

$du = -\sin x \cdot dx$

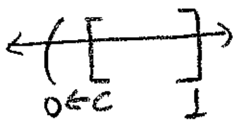
$(-1) \cdot du = \sin x \cdot dx$

$x = 0 \Rightarrow u = \cos 0 = 1$

$x = \frac{\pi}{2} \Rightarrow u = \cos \frac{\pi}{2} = 0$

$$= \int_1^0 \frac{1}{u^2} \cdot (-1) \cdot du$$

$$= \int_0^1 \frac{1}{u^2} \cdot du$$

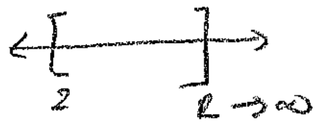


$$= \lim_{c \to 0^+} \int_c^1 u^{-2} \cdot du$$

$$= \lim_{c \to 0^+} \left(\frac{-1}{u} \right) \Big|_c^1 = \lim_{c \to 0^+} \left(-1 + \frac{1}{c} \right) = \infty$$

diverges to ∞.

(b) $\int_1^\infty x\, e^{-2x^2}\, dx$ $= \int_2^\infty e^{-u} \cdot \frac{1}{4} \cdot du$



$u = 2x^2$

$du = 4x \cdot dx$

$\frac{1}{4} \cdot du = x \cdot dx$

$x = 1 \Rightarrow u = 2$

$x \to \infty \Rightarrow u \to \infty$

$$= \frac{1}{4} \lim_{R \to \infty} \int_2^R e^{-u} \cdot du$$

$$= \frac{1}{4} \lim_{R \to \infty} \left(-e^{-u} \right) \Big|_2^R$$

$$= \frac{1}{4} \lim_{R \to \infty} \left(\frac{-1}{\cancel{e^R}_{0}} + \frac{1}{e^2} \right)$$

$$= \frac{1}{4e^2}$$

converges to $1/4e^2$

(4) (*a*) Two nonnegative numbers have sum 60. What are the numbers if the product of one of them and the square of the other is maximal?

$x \geq 0, y \geq 0$ and $x+y=60$

Maximize $x \cdot y^2$
subject to $x+y=60$

If $x+y=60$ then $y=60-x$

Let $f(x)=x\cdot(60-x)^2$. We will find max of $f(x)$.

$$f'(x)=(60-x)^2+x\cdot 2(60-x)\cdot(-1)$$
$$=(60-x)[60-x-2x]$$
$$f'(x)=(60-x)(60-3x)$$
$$f'(x)=0 \Rightarrow x=60 \;;\; x=20$$
$$x=60 \Rightarrow f(60)=0 \;;\; x=20 \Rightarrow y=40$$

(*b*) Find the length of the curve $y=\dfrac{x^3}{6}+\dfrac{1}{2x}$ from $x=1/2$ to $x=2$.

$$y=f(x)=\frac{x^3}{6}+\frac{1}{2x}$$
$$f'(x)=\frac{x^2}{2}-\frac{1}{2x^2}$$
$$1+(f'(x))^2=1+\frac{x^4}{4}-\frac{1}{2}+\frac{1}{4x^4}$$
$$=\frac{x^4}{4}+\frac{1}{2}+\frac{1}{4x^4}$$
$$=\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)^2$$

$$\sqrt{1+(f'(x))^2}=\sqrt{\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)^2}$$

length of the curve $=\displaystyle\int_{\frac{1}{2}}^{2}\sqrt{1+(f'(x))^2}\cdot dx$

$$=\int_{1/2}^{2}\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)dx$$
$$=\left(\frac{1}{2}\cdot\frac{x^3}{3}-\frac{1}{2x}\right)\Bigg|_{1/2}^{2}$$
$$=\frac{99}{48}$$