25 points	25 points	25 points	25 points	100 points
1	2	3	4	**Total**

MATH 153 CALCULUS I 22.01.2011

İzmir University of Economics Faculty of Arts and Science Department of Mathematics

FINAL EXAM

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1. Let P_n denote the partition of the interval $[0, 1]$ into n subintervals of equal length $\Delta x = 1/n$. For a given function $f(x) = 3 - 2x$ defined on $[0, 1]$

(a) Calculate $L(f, P_n)$.

$f(x) = 3-2X$ on $[0,1]$ $P_n = \{0, \frac{1}{n}, \frac{2}{n}, \dots, \frac{n-1}{n}, \frac{n}{n}\}$

we have

$$L(f, P_n) = \frac{1}{n}\cdot(3-2.0+3-2.\frac{1}{n}+3-2.\frac{2}{n}+\dots+3-2.(\frac{n-1}{n}))$$

$$= \frac{1}{n}\cdot(3n-2.(0+\frac{1}{n}+\frac{2}{n}+\dots+\frac{n-1}{n}))$$

$$= 3 - \frac{2}{n}\cdot\frac{1}{n}\cdot\frac{(n-1).n}{2} = 3 - \frac{n-1}{n} = \frac{2n+1}{n}$$

(b) Calculate $U(f, P_n)$

$$U(f, P_n) = \frac{1}{n}(3-2.\frac{1}{n}+3-2.\frac{2}{n}+\dots+3-2.\frac{n}{n})$$

$$= \frac{1}{n}(3n-2.(\frac{1}{n}+\frac{2}{n}+\dots+\frac{n}{n}))$$

$$= 3 - \frac{2}{n}\cdot\frac{1}{n}\cdot(1+2+\dots+n)$$

$$= 3 - \frac{2}{n^2}\cdot\frac{n.(n+1)}{2} = 3 - \frac{n+1}{n} = \frac{2n-1}{n}$$

(c) Show that $\lim_{n\to\infty} L(f, P_n) = \lim_{n\to\infty} U(f, P_n)$. What is $\int_0^1 (3 - 2x)dx$?

$$\lim_{n\to\infty} L(f, P_n) = 2$$

$$\lim_{n\to\infty} U(f, P_n) = 2$$

$$\int_0^1 (3-2x)dx = 3x - x^2 \Big|_0^1 = 3-1 = 2$$

2. (a) Sketch and find the area of the plane region bounded by the curves $x = y^2$ and $x = 2y^2 - y - 2$.

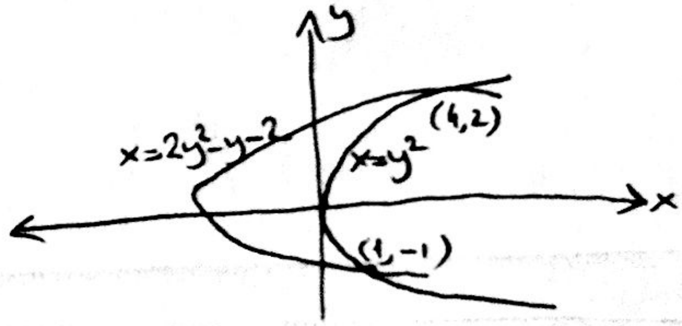
For intersections: $y^2 = 2y^2 - y - 2 \Rightarrow y^2 - y - 2 = 0$

$(y-2)(y+1) = 0 \Rightarrow y = -1$ or $y = 2$.

$$\text{Area of } R = \int_{-1}^{2} [y^2 - (2y^2 - y - 2)]\,dy$$

$$= \int_{-1}^{2} [2 + y - y^2]\,dy = \left(2y + \frac{1}{2}y^2 - \frac{1}{3}y^3\right)\Big|_{-1}^{2}$$

$= \frac{9}{2}$ sq. units



(b) Evaluate the integral. $\int x^3 cos(x^2)dx$

$$\int x^3 \cos(x^2)\,dx = \int x^2 . \cos(x^2) . x . dx$$

$x^2 = t \Rightarrow 2x\,dx = dt$

$\Rightarrow x . dx = \frac{1}{2} dt$

$$= \frac{1}{2} \int t . \cos t . dt$$

$t = u \quad \int \cos t\,dt = \int dv$

$dt = du \quad \sin t = v$

$$\Rightarrow \frac{1}{2}\left(t . \sin t - \int \sin t . dt\right)$$

$$= \frac{1}{2}(t . \sin t + \cos t) + C$$

3. Evaluate the integral.

(a) $\int \frac{x^3+1}{12+7x+x^2}dx$

$$\frac{x^3+1}{x^2+7x+12} = x-7+\frac{37x+85}{(x+4).(x+3)}$$

$$\frac{37x+85}{(x+4)(x+3)} = \frac{A}{x+4}+\frac{B}{x+3} = \frac{(A+B)x+3A+4B}{x^2+7x+12}$$

$$\Rightarrow \begin{cases} A+B=37 \\ 3A+4B=85 \end{cases} \Rightarrow A=63, \; B=-26$$

Now we have

$$\int \frac{x^3+1}{12+7x+x^2}dx = \int \left(x-7+\frac{63}{x+4}-\frac{26}{x+3}\right)dx$$

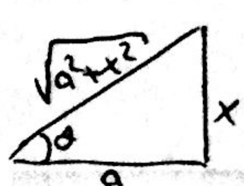
$$= \frac{x^2}{2}-7x+63\ln|x+4|-26\ln|x+3|+C$$

(b) $\int \frac{dx}{(a^2+x^2)^{3/2}}$ Let $x=a\tan\theta$

$dx = a\sec^2\theta \, d\theta$

$$= \int \frac{a.\sec^2\theta}{(a^2+a^2\tan^2\theta)^{3/2}}d\theta = \int \frac{a\sec^2\theta}{a^3\sec^3\theta}d\theta$$

$$= \frac{1}{a^2}\int \cos\theta \, d\theta = \frac{1}{a^2}\sin\theta + C = \frac{x}{a^2.\sqrt{a^2+x^2}}+C$$



4. Evaluate the given integral or show that it diverges.

(a) $\int_1^e \frac{dx}{x\sqrt{\ln x}}$

$$\int_1^e \frac{dx}{x\sqrt{\ln x}} \qquad \text{Let } u = \ln x$$

$$du = \frac{1}{x}dx$$

$$= \int_0^1 \frac{du}{\sqrt{u}} = \lim_{c \to 0^+} 2\sqrt{u}\Big|_c^1 = 2$$

(b) $\int_{-\infty}^{\infty} xe^{-x^2}dx = \underbrace{\int_{-\infty}^{0} x.e^{-x^2}dx}_{J_1} + \underbrace{\int_0^{\infty} x.e^{-x^2}dx}_{J_2}$

$$J = J_1 + J_2$$

$$J_2 = \int_0^{\infty} x.e^{-x^2}dx \qquad \text{Let } x^2 = u$$

$$2x\,dx = du \Rightarrow x.dx = \frac{1}{2}du$$

$$= \frac{1}{2}\int_0^{\infty} e^{-u}du = \frac{1}{2}\lim_{R\to\infty} -e^{-u}\Big|_0^R = \frac{1}{2}$$

Similarly, $J_1 = -\frac{1}{2}$

Therefore, $J = 0$