KEY

25 points	25 points	25 points	25 points	100 points
1	2	3	4	**Total**

MATH 153 CALCULUS I 31.10.2014

İzmir University of Economics Faculty of Arts and Sciences, Department of Mathematics

Midterm Exam 1

Student Name and Surname:.............................

Instructor's Name:...

(1) (a) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{\cancel{\sqrt{x}-3}}{(\cancel{\sqrt{x}-3})(\sqrt{x}+3)} = \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$

(b) $\lim_{x\to -8^+} \frac{2x}{x+8} = -\infty$

(c) $\lim_{x\to -1} \frac{\sqrt{x^2+8}-3}{x+1}$

$$\lim_{x\to -1} \frac{\sqrt{x^2+8}-3}{x+1} \cdot \frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3} = \lim_{x\to -1} \frac{x^2-1}{(x+1)(\sqrt{x^2+8}+3)}$$

$$= \lim_{x\to -1} \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(\sqrt{x^2+8}+3)} = \frac{-1}{3}$$

(d) $\lim_{x\to -\infty} \frac{x^2-4x+8}{3x^3}$

$$\lim_{x\to -\infty} \frac{\cancel{x^3}\left(\frac{1}{x} - \frac{4}{x^2} + \frac{8}{x^3}\right)}{3\cancel{x^3}} = \frac{0}{3} = 0$$

(e) $\lim_{x\to 3} \frac{|5-2x|-|x-2|}{|x-5|-|3x-7|}$

$$\lim_{x\to 3} \frac{-5+2x-x+2}{-x+5-3x+7} = \lim_{x\to 3} \frac{\cancel{x-3}}{-4\cancel{(x-3)}} = \frac{-1}{4}$$

(2) Calculate the derivatives of the following functions:

(a) $f(x) = \left(4 - x^{\frac{2}{5}}\right)^{\frac{-5}{2}}$

$$f'(x) = -\frac{\cancel{5}}{\cancel{2}}\left(4 - x^{\frac{2}{5}}\right)^{-\frac{7}{2}} \cdot \left(-\frac{\cancel{2}}{\cancel{5}} x^{-\frac{3}{5}}\right) = x^{-\frac{3}{5}} \cdot \left(4 - x^{\frac{2}{5}}\right)^{-\frac{7}{2}}$$

(b) $f(x) = \sqrt{2 + \cos^2 x}$

$$f'(x) = \frac{1}{2}\left(2 + \cos^2 x\right)^{-\frac{1}{2}} \cdot 2\cos x \cdot -\sin x$$

$$f'(x) = -\frac{\sin x \cdot \cos x}{\sqrt{2 + \cos^2 x}}$$

(c) $f(x) = \frac{\sqrt{x}}{x+1}$

$$f'(x) = \frac{\frac{1}{2}x^{-1/2} \cdot (x+1) - \sqrt{x}}{(x+1)^2} = \frac{\frac{x+1}{2\sqrt{x}} - \frac{\sqrt{x}}{1}_{(2\sqrt{x})}}{(x+1)^2}$$

$$= \frac{x+1-2x}{2\sqrt{x} \cdot (x+1)^2} = \frac{-x+1}{2\sqrt{x}\,(x+1)^2}$$

(3) (*a*) Find the equation of the tangent to the curve $\cos\left(\frac{\pi y}{x}\right) = \frac{x^2}{y} - \frac{17}{2}$ at $(3, 1)$.

$$-\sin\left(\frac{\pi y}{x}\right) \cdot \pi\left(\frac{y' \cdot x - 1 \cdot y}{x^2}\right) = \frac{2x \cdot y - y' \cdot x^2}{y^2}$$

$$-\sin\left(\frac{\pi}{3}\right) \cdot \pi\left(\frac{3y' - 1}{9}\right) = 6 - 9y'$$

$$-\sin\left(\frac{\pi}{3}\right)\left(\frac{\pi y'}{3} - \frac{\pi}{9}\right) = 6 - 9y'$$

$$-\frac{\sqrt{3}}{2} \cdot \pi \cdot \frac{y'}{3} + \frac{\sqrt{3}}{2} \cdot \frac{\pi}{9} = 6 - 9y'$$

$$-\frac{\sqrt{3}}{6}\pi y' + 9y' = 6 - \frac{\sqrt{3}\pi}{18}$$

$$y'\left(\frac{54 - \sqrt{3}\pi}{6}\right) = \frac{108 - \sqrt{3}\pi}{18}$$

$$y' = \frac{108 - \sqrt{3}\pi}{3(54 - \sqrt{3}\pi)}$$

$$y = m \cdot (x - x_0) + y_0$$

$$y = \frac{108 - \sqrt{3}\pi}{3(54 - \sqrt{3}\pi)} \cdot (x - 3) + 1 \Rightarrow \text{eqn. of tangent line}$$

(*b*) Calculate the derivative of the function $f(x) = \frac{1}{\sqrt{1+x}}$ directly from the definition of derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad f(x+h) = \frac{1}{\sqrt{1+x+h}} \qquad f(x) = \frac{1}{\sqrt{1+x}}$$

$$\lim_{h \to 0} \frac{\frac{1}{\sqrt{1+x+h}} - \frac{1}{\sqrt{1+x}}}{h} = \lim_{h \to 0} \frac{\frac{\sqrt{1+x} - \sqrt{1+x+h}}{\sqrt{1+x} \cdot \sqrt{1+x+h}} \cdot \frac{\sqrt{1+x} + \sqrt{1+x+h}}{\sqrt{1+x} + \sqrt{1+x+h}}}{h}$$

$$\lim_{h \to 0} \frac{1}{\cancel{h}} \cdot \frac{\cancel{1} + \cancel{x} - \cancel{1} - \cancel{x} - \cancel{h}}{\sqrt{1+x} \cdot \sqrt{1+x+\cancel{h}}\left(\sqrt{1+x} + \sqrt{1+x+\cancel{h}}\right)} = \lim_{h \to 0} \frac{-1}{2(1+x)(1+x)^{1/2}}$$

$$= \frac{-1}{2(1+x)^{3/2}}$$

(4) (*a*) Calculate enough derivatives of the function $f(x) = \dfrac{1}{(2x+1)^2}$ to enable you to guess the general formula for $f^{(n)}(x)$.

$f(x) = (2x+1)^{-2}$

$f'(x) = -2(2x+1)^{-3} \cdot 2$

$f''(x) = 2.3.(2x+1)^{-4} \cdot 2\,2$

$f'''(x) = -2.3.4.(2x+1)^{-5} \cdot 2\,2\,2$

$\vdots$

$f^{n}(x) = (-1)^{n} \cdot (n+1)! \cdot 2^{n} \cdot (2x+1)^{-(n+2)}$

(*b*) For what values of a is

$$f(x) = \begin{cases} x^2 - 1 & x < 3 \\ 2ax & x \geq 3 \end{cases}$$

continuous at every x?

$$\lim_{x \to 3^+} 2ax = \lim_{x \to 3^-} (x^2 - 1)$$

$6a = 8$

$a = \dfrac{4}{3}$