MATH 103 – HOMEWORK ASSIGNMENT I

1. Consider the predicate $x = 2y$ which contains two free variables x, y of integers. There are only *six different ways* to use quantification to turn this predicate into a statement (why six but not four or eight, understand this firstly). Find all of these six statements and determine the truth or falsehood of each.

2. Write the negation of the following statements:

a. For every positive real number ε, there exists a positive real number δ such that for all real numbers x, if $|x - 3| < \delta$, then $|(x^2 - 2) - 7| < \varepsilon$.

b. For all integers x, if x is an odd integer, then x^2 is an odd integer.

c. For every positive real number a, there exists a positive real number x such that $x^2 = a$.

3. Reformulate each of the following theorems in the form $p \to q$. (The statements of the theorems as given below are commonly used in mathematics courses, they are not necessarily the best possible ways to state these theorems.)

a. The area of the region inside a circle of radius r is πr^2.

b. Given a line ℓ and a point P not on ℓ, there exists exactly one line ℓ' containing P that is parallel to ℓ.

c. (Fundamental Theorem of Calculus) Let f be a continuous function on $[a, b]$, and let F be any function for which $F'(x) = f(x)$. Then

$$\int_a^b f(x)dx = F(b) - F(a).$$

4. Remember that for a real number x, the *absolute value* of x is defined by

$$|x| = \begin{cases} x, & if x \geq 0 \\ -x, & if x < 0 \end{cases}$$

Let a and b be real numbers. Prove the following statements:

a. $|-a| = |a|$.

b. $|a|^2 = a^2$.

c. $|ab| = |a||b|$.

5. Remember that for two integers a and b, not both zero, their *greatest common divisor* $\gcd(a, b)$ can be expressed as a *linear combination* of a and b:

$$\gcd(a, b) = sa + tb \text{ for some } s, t \in \mathbb{Z}.$$

Such s and t can be found using *Euclidean Algorithm.* Also recall that two nonzero integers a and b are called *relatively prime* if $\gcd(a, b) = 1$.

Use these facts to prove that for $n \in \mathbb{Z}^+$ and $a \in \mathbb{Z}$, there exists a' satisfying $a'a \equiv 1 (\mod n)$ if and only if a and n are *relatively prime.*

Due Date: October 31, 2014 (at 12:00).