IUE – MATH 103 – Fundamentals of Mathematics

1st Midterm — November 5, 2014 — 11:00 - 12:50

Name: ______________________ **ID #:** ______________________

Q1	Q2	Q3	Q4	TOTAL
25	25	25	25	100

Important: Show all your work. Answers without sufficient explanation might not get full credit. Be neat.

Signature: ______________________

GOOD LUCK!

1. Briefly explain the following terms:
a) commutative (operation),
b) associative (operation),
c) well ordering principle,
d) tautology,
e) contrapositive.

Solution:

a) $+$ is a commutative *binary operation* on $\mathbb{R}$ since for all $a, b \in \mathbb{R}$,

$$a + b = b + a.$$

$\cdot$ is a commutative *binary operation* on $\mathbb{R}$ since for all $a, b \in \mathbb{R}$,

$$a \cdot b = b \cdot a.$$

b) $+$ is a associative *binary operation* on $\mathbb{R}$ since for all $a, b, c \in \mathbb{R}$,

$$(a + b) + c = a + (b + c).$$

$\cdot$ is a associative *binary operation* on $\mathbb{R}$ since for all $a, b, c \in \mathbb{R}$,

$$(a \cdot b) \cdot c = a \cdot (b \cdot c).$$

c) WOP: Any nonempty subset of whole numbers has a smallest element.
d) A statement whose truth table values are all true is called a tautology.
e) The contrapositive of a statement $p \to q$ is defined as $\neg q \to \neg p$.

2. Take the negation of the following compound statements.

a) There is a function whose graph intersects the x-axis.

b) $\forall a \in \mathbb{R},\ \exists! b \in \mathbb{R}\ :\ a + b = 0.$

Solution:

a) The statement is

$p :$ There is a function f such that $f(x) = 0$ for some $x \in \mathbb{R}$.

The negation of the statement is

$\neg p :$ For all function f and for all $x \in \mathbb{R}$, $f(x) \neq 0$.

b) The statement is

$p : (\forall a \in \mathbb{R},\ \exists b \in \mathbb{R}\ :\ a + b = 0) \wedge [(\forall a \in \mathbb{R},\ \exists b, b' \in \mathbb{R}\ :\ a + b = 0 \wedge a + b' = 0) \rightarrow b = b']$.

The negation of the statement is

$\neg p : (\exists a \in \mathbb{R}\ :\ \forall b \in \mathbb{R},\ a + b \neq 0) \vee (\forall a \in \mathbb{R},\ \exists b, b' \in \mathbb{R}\ :\ b \neq b'.\ a + b = 0,\ a + b' = 0)$

3. a) Find the greatest common divisor of 5767 and 5609.
b) Find $m, n \in \mathbb{Z}$ such that $181m + 113n = 1$.

Solution:

a) By Euclidean algorithm,

$$\begin{aligned} 5767 &= 1 \cdot 5609 + 158 \\ 5609 &= 35 \cdot 158 + 79 \\ 158 &= 2 \cdot 79 + 0. \end{aligned}$$

the last nonzero remainder 79 is $\gcd(5767, 5609)$.

b) By Euclidean algorithm, we have

$$\begin{aligned} 181 &= 1 \cdot 113 + 68 \\ 113 &= 1 \cdot 68 + 45 \\ 68 &= 1 \cdot 45 + 23 \\ 45 &= 1 \cdot 23 + 22 \\ 23 &= 1 \cdot 22 + 1 \\ 22 &= 22 \cdot 1 + 0. \end{aligned}$$

Then

$$\begin{aligned} 68 &= 181 - 113 \\ 45 &= 113 - 68 = 2.113 - 181 \\ 23 &= 68 - 45 = 2 \cdot 181 - 3 \cdot 113 \\ 22 &= 45 - 23 = 5 \cdot 113 - 3 \cdot 181 \\ 1 &= 23 - 22 = 5 \cdot 181 - 8 \cdot 113. \end{aligned}$$

Hence $m = 5$ and $n = -8$.

4. Prove or disprove:

$$\text{If } p \text{ is stronger than } q, \text{ then } q \to r \text{ is stronger than } p \to r.$$

Solution: Suppose that p is stronger than q. Then $p \to q$ is a tautology while $q \to p$ is not. Since $\neg q \to \neg p$ is equivalent to $p \to q$, $\neg q \to \neg p$ is a tautology while $\neg p \to \neg q$ is not. So, $\neg q$ is stronger than $\neg p$. Therefore, $\neg q \vee r$ is stronger than $\neg p \vee r$. Since $\neg p \vee r$ is equivalent to $p \to r$, we obtain that $q \to r$ is stronger than $p \to r$.