IUE – MATH 103 – Fundamentals of Mathematics

1st Midterm — November 7, 2013 — 16:00 - 17:50

Name: ______________________ **ID #:** ______________________

Q1	Q2	Q3	Q4	TOTAL
25	25	25	25	100

Important: Show all your work. Answers without sufficient explanation might not get full credit. Be neat.

Signature: ______________________

GOOD LUCK!

1. Compare the following statements and conclude whether they are equivalent or one of them is stronger than the other.

a) $(p \to q) \wedge (q \to r)$ $(p \to r)$,

b) $(p \wedge q) \to r$ $(p \wedge \neg r) \to \neg q$.

Solution:

a) If $(p \to q) \wedge (q \to p)$ is TRUE, then both $p \to q$ and $q \to p$ are TRUE. Therefore, whenever p is TRUE, q must be TRUE, and hence r must be TRUE. Thus $p \to r$ is TRUE. I.e., $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology showing that the first compound statement is STRONGER. Moreover, this strength is strict since when p & r are TRUE and q is FALSE, the stronger statement becomes FALSE but the weaker is TRUE. Hence, they are not equivalent.

b) $p \wedge q \to r \Leftrightarrow \neg(p \wedge q) \vee r$

$\Leftrightarrow \neg p \vee \neg q \vee r \Leftrightarrow (\neg p \vee r) \vee \neg q$

$\Leftrightarrow \neg(p \wedge \neg r) \wedge \neg q \Leftrightarrow (p \wedge \neg r) \to \neg q$

$\therefore$They are Equivalent.

2. Take the negation of the following compound statements.

a) $(\exists m \in A)(\forall x \in A)(x \leq m)$,

b) There exist $M_1, M_2 \in \mathbb{R}$ such that $M_1 \leq x \leq M_2$ for all $x \in S$.

Solution:

a) $(\forall m \in A)(\exists x \in A)(x > m)$

b) $(\forall M_1, M_2 \in R)(\exists x \in S)(x < M_1 \vee x > M_2)$

3. a) Write the definition of a greatest common divisor of two nonzero integers.
b) Find $m, n \in \mathbb{Z}$ such that $33m + 35n = 1$.

Solution:

a) Let a and b be two non zero integers. A positive integer d is called gcd if

i)$d|a \wedge d|b$
ii)$e|a \wedge e|b \to e|d$

b) $35 = 1.33 + 2$
$33 = 16.2 + 1 \to \text{gcd} = 1$
$1 = 33 - 16.2 = 33 - 16(35 - 33) = 17.33 - 16.35$
Thus $m = 17$, $n = -16$.

4. Prove or disprove:
For any sets A, B, C, we have,

$$A \cap C = B \cap C \ \wedge \ A \cup C = B \cup C \quad \rightarrow \quad A = B.$$

Solution:
We will prove the result.
$A \subseteq B$:
Let $x \in A$. Then $x \in A \cup C = B \cup C \rightarrow x \in B \cup C$
if $x \in B$, we are done.
if $x \in C$, then $x \in A$ and $x \in C$
$\rightarrow x \in A \cap C = B \cap C \rightarrow x \in B \cap C$
$\rightarrow x \in B$. Hence $A \subseteq B$.
$B \subseteq A$: Similar to above.