= KEY =

25 points	25 points	25 points	25 points	100 points
1	2	3	4	**Total**

MATH 102 CALCULUS II 04.04.2016

İzmir University of Economics Faculty of Arts and Sciences, Department of Mathematics

FIRST MIDTERM EXAM

Student Name:...

Student Number:...

Department:...

Section: Check for your instructor below:

- [ ] Baruch Schneider
- [ ] Ahmet Genç
- [ ] Demet Ersoy Özdek
- [ ] Aslı Güldürdek

7/ 1. Evaluate the given integrals:

(a) $\int_1^2 (3x^2 + 2x - \frac{7}{\sqrt{x}})\, dx = \int_1^2 (3x^2 + 2x - 7x^{-1/2})\, dx$

$= x^3 + x^2 - \frac{7x^{1/2}}{1/2} \Big|_1^2$

$= (8 + 4 - 14\sqrt{2}) - (1 + 1 - 14)$

$= 12 - 14\sqrt{2} + 12$

$= \boxed{24 - 14\sqrt{2}}$

9/ (b) $\int_1^3 (\frac{1}{x} - e^{-3x})\, dx = \ln x - \frac{e^{-3x}}{-3} \Big|_1^3$

$= \ln x + \frac{1}{3e^{3x}} \Big|_1^3$

$= \boxed{\left(\ln 3 + \frac{1}{3e^9}\right) - \left(\frac{1}{3e^3}\right)}$

9/ (c) $\int_{-1}^{1} x^4(2 + x^5)\, dx$ $\quad u = 2 + x^5$

$du = 5x^4 dx \mapsto x^4 dx = \frac{du}{5}$.

$\int u \cdot \frac{du}{5} = \frac{1}{5}\int u\, du = \frac{1}{10}(2 + x^5)^2 \Big|_{-1}^{1} = \frac{9}{10} - \frac{1}{10} = \boxed{\frac{4}{5}}$

= OR =

$\int (2x^4 + x^9) dx = \frac{2}{5}x^5 + \frac{x^{10}}{10} \Big|_{-1}^{1} = \left(\frac{2}{5} + \frac{1}{10}\right) - \left(-\frac{2}{5} + \frac{1}{10}\right)$

$= \frac{5}{10} - \left(-\frac{3}{10}\right) = \frac{8}{10} = \boxed{\frac{4}{5}}$

12/ 2. (a) Find the particular solution for the differential equation:
$$\frac{dy}{dx} = e^x - 4x \quad ; \; y(0) = 5$$

$$dy = (e^x - 4x)dx$$

$$\int dy = \int (e^x - 4x)dx$$

$$y = e^x - \frac{4x^2}{2} + C_1 = e^x - 2x^2 + C_1 .$$

$$y(0) = 1 - 0 + C_1 = 5$$

$$\boxed{C_1 = 4}$$

$$\boxed{y(x) = e^x - 2x^2 + 4}$$

☒

13/ (b) The marginal cost function for producing x cell phones per day is given by $C'(x) = \dfrac{1}{\sqrt{2x+16}}$; $C(0) = 14$.
Find the cost function $C(x)$.

$$C(x) = \int \frac{dx}{\sqrt{2x+16}}$$

$$u = 2x + 16$$

$$du = 2dx \mapsto dx = \frac{du}{2} .$$

$$\int \frac{du/2}{\sqrt{u}} = \frac{1}{2} \int u^{-1/2} du = \frac{1}{2} \frac{u^{1/2}}{1/2} + K$$

$$C(x) = \sqrt{2x+16} + K$$

$$C(0) = 4 + K = 14$$

$$\boxed{K = 10}$$

$$\boxed{C(x) = \sqrt{2x+16} + 10}$$

☒

3. Evaluate the following integrals:

8/ (a) $\int_0^1 \frac{(x+3)}{(x^2+6x+8)}\, dx$

$u = x^2+6x+8$

$du = (2x+6)dx \mapsto (x+3)dx = \frac{du}{2}$

$\int \frac{du/2}{u} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|x^2+6x+8| \Big|_0^1$

$= \boxed{\frac{1}{2}(\ln 15 - \ln 8)}$ ⊠

7/ (b) $\int 2(2x-6)^9\, dx$

$u = 2x-6$

$du = 2dx$

$\int u^9 du = \frac{u^{10}}{10} + C = \boxed{\frac{(2x-6)^{10}}{10} + C}$ ⊠

10/ (c) $\int_1^e (x^4 \ln x)\, dx$

$u = \ln x \rightarrow du = \frac{dx}{x}$

$dv = x^4 dx \rightarrow v = \frac{x^5}{5}$

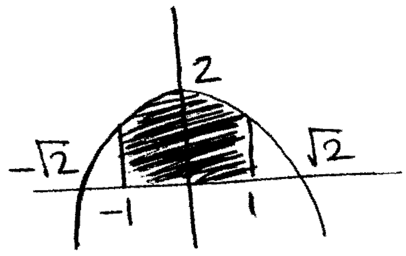
$\Rightarrow = \ln x \cdot \frac{x^5}{5} - \int \frac{x^5}{5}\frac{dx}{x}$

$= \ln x \frac{x^5}{5} - \int \frac{x^4}{5} dx = \ln x \cdot \frac{x^5}{5} - \frac{x^5}{25} \Big|_1^e$

$= \left(\ln e \frac{e^5}{5} - \frac{e^6}{30}\right) - \left(-\frac{1}{25}\right) = \boxed{\frac{e^5}{5} - \frac{e^6}{30} + \frac{1}{25}}$ ⊠

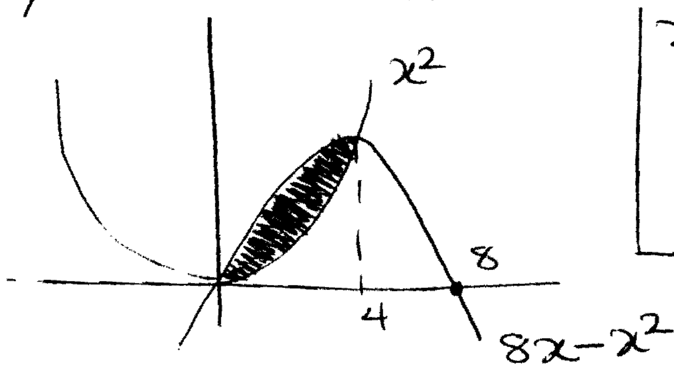
$$2 - x^2 = 0$$
$$x^2 = 2 \mapsto x = \pm\sqrt{2}.$$

12/ 4. (a) Find the area bounded by $f(x) = 2 - x^2$ and $y = 0$ for $-1 \le x \le 1$



$$A = \int_{-1}^{1} (2 - x^2)\,dx$$
$$= \left(2x - \frac{x^3}{3}\right)\Big|_{-1}^{1}$$
$$= \left(2 - \frac{1}{3}\right) - \left(-2 + \frac{1}{3}\right)$$
$$= 4 - \frac{2}{3} = \boxed{\frac{10}{3}}$$

13/ (b) $f(x) = x^2$ and $g(x) = 8x - x^2$ are given. Find the area bounded by f and g.



$$x^2 = 8x - x^2$$
$$2x^2 = 8x$$
$$2x(x-4) = 0.$$
$$x = 0 \qquad x = 4$$

$$8x - x^2 = 0$$
$$x = 0 \quad x = 8$$

$$A = \int_0^4 \left[(8x - x^2) - x^2\right] dx = \int_0^4 (8x - 2x^2)\,dx$$
$$= 8\frac{x^2}{2} - 2\frac{x^3}{3}\Big|_0^4$$
$$= 4x^2 - \frac{2}{3}x^3\Big|_0^4$$
$$= 64 - \frac{128}{3} = \boxed{\frac{64}{3}}$$