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25 points	25 points	25 points	25 points	100 points
1	2	3	4	**Total**

MATH 102 CALCULUS II 13.05.2016

İzmir University of Economics Faculty of Arts and Sciences, Department of Mathematics

SECOND MIDTERM EXAM

Student Name: .....

Student Number: .....

Department: .....

Section: **Check for your instructor below:**

- ☐ Baruch Schneider
- ☐ Ahmet Genç
- ☐ Demet Ersoy Özdek
- ☐ Aslı Güldürdek

5/ 1. (a) The annual cost for producing cell-phones is given by:

$$C(x, y) = 2x^2 + 2xy + 3y^2 - 16x - 18y + 54.$$

where x is the amount spent per year on labor and y is the amount spent per year on equipment(both in millions of dollars).
What should x and y be for a minimum cost?

$C_x = 4x + 2y - 16 = 0 \longrightarrow 2x + y = 8$

$C_y = 2x + 6y - 18 = 0 \qquad x + 3y = 9 \;/-2$

$-5y = -10 \longrightarrow \boxed{y=2} \; \boxed{x=3}$

CHECK $A = C_{xx} = 4$, $C = C_{yy} = 6$, $B = C_{xy} = 2$ } $AC - B^2 = 24 - 4 = 20 > 0$, $A = 4 > 0$ } l. min

(b) $f(x, y) = xy^3 + 5xy^2 + 2x + 1$ is given. Evaluate the following:

5/ i. $f_x = y^3 + 5y^2 + 2$

5/ ii. $f_x(0,1) = y^3 + 5y^2 + 2 \Big|_{(0,1)} = 1 + 5 + 2 = \boxed{8}$

5/ iii. $f_{xy} = 3y^2 + 10y$

5/ iv. $f_{xy}(1,0) = 3y^2 + 10y \Big|_{(1,0)} = \boxed{0}$

12/ 2. (a) Find and classify the local extrema for the function: $f(x,y) = x^2 - 6xy - 2y^3$.

$x=0$ $x=-9$

$y=0$ $y=-3$

$f_x = 2x-6y=0 \longrightarrow x=3y$

$f_y = -6x-6y^2=0 \rightarrow -18y-6y^2=0 \rightarrow 6y(-3-y)=0$

$A = f_{xx} = 2$
$C = f_{yy} = -12y$
$B = f_{xy} = -6$

$AC-B^2 = -24y-36$

(0,0) $AC-B^2 = -36<0 \longrightarrow$ saddle point.

(-9,-3) $AC-B^2 = 180>0$, $A = 2>0 \longrightarrow$ l. minimum

13/ (b) Use Lagrange multipliers method to find the minimum value of $f(x,y) = x^2 + 2y^2 - xy$ subject to $2x + y = 22$.

$F(x,y,\lambda) = x^2+2y^2-xy+\lambda(2x+y-22)$

$F_x = 2x-y+2\lambda = 0 \rightarrow 2x-y+2\lambda = 0$

$F_y = 4y-x+\lambda = 0 \rightarrow 8y-2x+2\lambda = 0$

$F_\lambda = 2x+y-22 = 0$

$4x-9y=0$

$x = \frac{9}{4}y$

$2\cdot\frac{9}{4}y+y = 22$

$22y = 4\cdot 22$

$y=4$ $x=9$ $\lambda=-7$

$f(9,4) = 81+2\cdot 16-9\cdot 4$

$= 81+32-36$

$= 77$

3. Evaluate the indicated integrals:

12/ (a) $\int_0^1 \int_1^2 (x^2y - 2x)\, dxdy.$

$$= \int_0^1 \left(\frac{x^3}{3}y - x^2 \Big|_1^2 \right) dy$$

$$= \int_0^1 \left[\left(\frac{8}{3}y - 4 \right) - \left(\frac{1}{3}y - 1 \right) \right] dy$$

$$= \int_0^1 \left(\frac{7}{3}y - 3 \right) dy$$

$$= \frac{7}{3}\frac{y^2}{2} - 3y \Big|_0^1 = \frac{7}{6} - 3 = \boxed{\frac{-11}{6}}$$

13/ (b) $\int_0^1 \int_x^{2x} e^{y-x}\, dydx =$

$$= \int_0^1 \left(e^{-x} \int_x^{2x} e^y\, dy \right) dx$$

$$= \int_0^1 e^{-x} \left(e^y \Big|_x^{2x} \right) dx = \int_0^1 e^{-x} \left(e^{2x} - e^x \right) dx$$

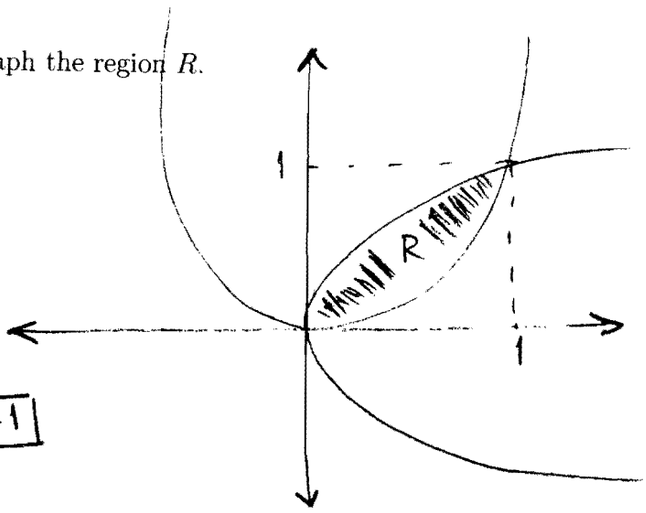
$$= \int_0^1 \left(e^x - e^0 \right) dx = e^x - x \Big|_0^1$$

$$= (e - 1) - (e^0 - 0)$$

$$= \boxed{e - 2}$$

4. The region R is bounded by $y = x^2$, $x = y^2$.

5/ (a) Graph the region R.



$x^2 = \sqrt{x}$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$\boxed{x=0}$ $\boxed{x=1}$

5/ (b) Describe the region R.

$$R_x = \{(x,y) \mid x^2 \le y \le \sqrt{x}\,;\ 0 \le x \le 1\}$$

$$= \text{OR} =$$

$$R_y = \{(x,y) \mid y^2 \le x \le \sqrt{y}\,;\ 0 \le y \le 1\}$$

15/ (c) Evaluate: $\iint_R 3x^2y\,dA$.

R_x

$$= 3\int_0^1 \int_{x^2}^{\sqrt{x}} x^2 y\,dy\,dx$$

$$= 3\int_0^1 x^2 \frac{y^2}{2}\Big|_{x^2}^{\sqrt{x}} dx$$

$$= \frac{3}{2}\int_0^1 \left(x^2 \cdot x - x^2 \cdot x^4\right) dx$$

$$= \frac{3}{2}\int_0^1 (x^3 - x^6)\,dx$$

$$= \frac{3}{2}\left(\frac{x^4}{4} - \frac{x^7}{7}\right)\Big|_0^1 = \frac{3}{2}\left(\frac{1}{4} - \frac{1}{7}\right) = \boxed{\frac{9}{56}}$$

R_y

$$= 3\int_0^1 \int_{y^2}^{\sqrt{y}} x^2 y\,dx\,dy$$

$$= 3\int_0^1 y\frac{x^3}{3}\Big|_{y^2}^{\sqrt{y}} dy$$

$$= \int_0^1 \left(y\,y^{3/2} - y\,y^6\right) dy$$

$$= \int_0^1 \left(y^{5/2} - y^7\right) dy$$

$$= \frac{2}{7}y^{7/2} - \frac{y^8}{8}\Big|_0^1 = \left(\frac{2}{7} - \frac{1}{8}\right) = \boxed{\frac{9}{56}}$$