25 points	25 points	25 points	25 points	100 points
1	2	3	4	Total

MATH 102 CALCULUS II 11.05.2015

İzmir University of Economics Faculty of Arts and Sciences, Department of Mathematics

SECOND MIDTERM EXAM

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10/

1. (a) The profit function for sales of two models of TV sets at a chain store is given by:

$$P(x,y) = \text{[illegible]}x + \text{[illegible]}y - 6x^2 + 4xy - 8y^2 - 500.$$

where x is the number of sales per week of model A and y is the number of sales per week of model B.
Evaluate $P_y(15,10)$

$P_y = 160 + 4x - 16y$

$P_y(15,10) = 160 + 4 \cdot 15 - 16 \cdot 10$

$= 160 + 60 - 160$

$= 60$ ■

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(b) Find and classify the local extrema for the function:

$$f(x,y) = 3y^2 - 2y^3 - 3x^2 + 6xy.$$

$f_x = -6x + 6y = 0 \longrightarrow x = y$

$f_y = 6y - 6y^2 + 6x = 0 \longrightarrow 6y - 6y^2 + 6y = 0$

$12y - 6y^2 = 0$

$6y(2-y) = 0$

$\boxed{\begin{matrix} y=0 \\ x=0 \end{matrix}}$ $\boxed{\begin{matrix} y=2 \\ x=2 \end{matrix}}$

$\begin{cases} A = f_{xx} = -6 \\ C = f_{yy} = 6 - 12y \\ B = f_{xy} = 6 \end{cases}$

$\underline{(0,0)}$ $AC - B^2 = (-6)(6-12y) - 6^2$

$= (-6)(6) - 36 = -72 < 0$

$\mapsto$ $(0,0)$ is a saddle point

$\underline{(2,2)}$ $AC - B^2 = (-6)(6-12y) - 6^2$

$= (-6)(6-24) - 36 = 72 > 0$

$A = -6 < 0$

$\longrightarrow$ $(2,2)$ is a l. maximum.

$f(x,y)$ takes a l.max value of 8 at $(2,2)$ ■

12/

2. (a) Use Lagrange multipliers method to maximize $f(x,y) = 49 - x^2 - y^2$ subject to $x + 3y = 10$.

$$F(x,y,\lambda) = 49 - x^2 - y^2 + \lambda(x+3y-10)$$

$$F_x = -2x + \lambda = 0 \mapsto \lambda = 2x$$

$$F_y = -2y + 3\lambda = 0 \mapsto -2y + 6x = 0 \mapsto y = 3x$$

$$F_\lambda = x + 3y - 10 = 0$$

$$x + 9x - 10 = 0$$

$$x = 1 \quad y = 3 \quad \lambda = 2.$$

$$F(1,3) = 49 - 1 - 9 = 39$$

13/

(b) Find the average value of $f(x,y) = 2 - 4x + 2y$ over the rectangle $R = \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 2\}$

$$\frac{1}{(1-0)(2-0)} \int_0^2 \int_0^1 (2-4x+2y)\,dx\,dy$$

$$= \frac{1}{2} \int_0^2 \left(2x - 2x^2 + 2yx \Big|_0^1 \right) dy$$

$$= \frac{1}{2} \int_0^2 (2 - 2 + 2y)\,dy$$

$$= \frac{1}{2} \left(y^2\right)\Big|_0^2 = \frac{1}{2} \cdot 4 = 2$$

$$\frac{1}{(1-0)(2-0)} \int_0^1 \int_0^2 (2-4x+2y)\,dy\,dx$$

$$= \frac{1}{2} \int_0^1 \left(2y - 4xy + y^2 \Big|_0^2 \right) dx$$

$$= \frac{1}{2} \int_0^1 (4 - 8x + 4)\,dx$$

$$= \frac{1}{2} \left(8x - 4x^2\right)\Big|_0^1 = \frac{1}{2} \cdot 4 = 2$$

3. Evaluate the indicated integrals:

12/ (a) $\int_0^3 \int_0^2 (x+2y)\,dx\,dy.$

$$= \int_0^3 \left(\frac{x^2}{2} + 2yx\right)\Big|_0^2 dy$$

$$= \int_0^3 (2+4y)\,dy$$

$$= 2y + 2y^2 \Big|_0^3$$

$$= 6 + 18 = 24 \quad \blacksquare$$

13/ (b) $\int_2^3 \int_1^2 xe^{xy}\,dy\,dx$

$$= \int_2^3 \left(\frac{x\,e^{xy}}{x}\right)\Big|_1^2 dx$$

$$= \int_2^3 \left(e^{2x} - e^{x}\right) dx$$

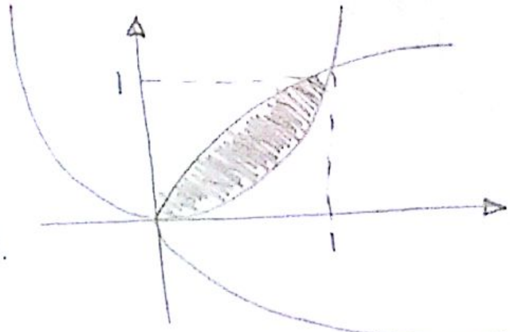
$$= \frac{e^{2x}}{2} - e^{x} \Big|_2^3$$

$$= \left(\frac{e^6}{2} - e^3\right) - \left(\frac{e^4}{2} - e^2\right) \quad \blacksquare$$

$x^2 = \sqrt{x}$

$x^4 = x \implies x(x^3 - 1) = 0$

$x = 0 \qquad x = 1$



13/ **4.** (a) The region R is bounded by $y = x^2$, $x = y^2$. Evaluate:
$$\iint_R xy^2 \, dA.$$

$R = \{(x,y) \mid x^2 \le y \le \sqrt{x},\ 0 \le x \le 1\}$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} xy^2 \, dy \, dx$$
$$= \int_0^1 \left(\frac{x}{3} y^3 \Big|_{x^2}^{\sqrt{x}} \right) dx$$
$$= \int_0^1 \left(\frac{x \cdot x\sqrt{x}}{3} - \frac{x \cdot x^6}{3} \right) dx$$
$$= \int_0^1 \left(\frac{x^{5/2}}{3} - \frac{x^7}{3} \right) dx$$
$$= \frac{2x^{7/2}}{21} - \frac{x^8}{24} \Big|_0^1 = \frac{2}{21} - \frac{1}{24} = \frac{9}{168}$$
$$= \frac{3}{56} \blacksquare$$

$R = \{(x,y) \mid y^2 \le x \le \sqrt{y},\ 0 \le y \le 1\}$

$$= \int_0^1 \int_{y^2}^{\sqrt{y}} xy^2 \, dx \, dy$$
$$= \int_0^1 \left(\frac{x^2}{2} y^2 \Big|_{y^2}^{\sqrt{y}} \right) dy$$
$$= \int_0^1 \left(\frac{y \cdot y^2}{2} - \frac{y^4 \cdot y^2}{2} \right) dy$$
$$= \int_0^1 \left(\frac{y^3}{2} - \frac{y^6}{2} \right) dy$$
$$= \frac{y^4}{8} - \frac{y^7}{14} \Big|_0^1 = \frac{1}{8} - \frac{1}{14} = \frac{6}{112}$$
$$= \frac{3}{56} \blacksquare$$

12/ (b) Evaluate $\iint_R \frac{1}{xy} \, dA$, where $R = \{(x,y) \mid 3 \le y \le 4, 1 \le x \le 2\}$

$$= \int_3^4 \int_1^2 \frac{1}{xy} \, dx \, dy$$
$$= \int_3^4 \left(\frac{1}{y} \ln x \Big|_1^2 \right) dy$$
$$= \int_3^4 \frac{1}{y} \ln 2 \, dy$$
$$= \ln 2 \cdot \int_3^4 \frac{1}{y} \, dy$$
$$= \ln 2 \cdot \left(\ln y \Big|_3^4 \right)$$

$$= \int_1^2 \int_3^4 \frac{1}{xy} \, dy \, dx$$
$$= \int_1^2 \left(\frac{1}{x} \ln y \Big|_3^4 \right) dx$$
$$= \int_1^2 \frac{1}{x} (\ln 4 - \ln 3) \, dx$$
$$= (\ln 4 - \ln 3) \int_1^2 \frac{1}{x} \, dx$$
$$= (\ln 4 - \ln 3) \left(\ln x \Big|_1^2 \right)$$
$$= (\ln 4 - \ln 3) \cdot \ln 2$$