KEY

25 points	25 points	25 points	25 points	100 points
1	2	3	4	**Total**

MATH 102 CALCULUS II 13.05.2014

İzmir University of Economics Faculty of Arts and Sciences, Department of Mathematics

SECOND MIDTERM EXAM

Student Name:...

Student Number:...

Department: ....

Section: **Check for your instructor below:**

☐ Yılmaz Akyıldız

☐ Baruch Schneider

☐ Ahmet Genç

☐ Aslı Güldürdek

1. (a) The annual labor and automated equipment cost (in millions of dollars) for a company's production of television sets is given by

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$$C(x, y) = 2x^2 + 2xy + 3y^2 - 16x - 18y + 54,$$

where x is the amount spent per year on labor and y is the amount spent per year on automated equipment (both in millions of dollars).
Evaluate $C_x(2, 3)$

$$C_x(x,y) = 4x + 2y - 16$$

$$C_x(2,3) = 8 + 6 - 16$$

$$= -2 \blacksquare$$

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(b) Find the volume of the solid under the graph of $f(x, y) = 3y^2 - x^2 + 2$ over the region $R = \{(x, y) \mid -1 \le x \le 1,\ 1 \le y \le 2\}$

$$V = \int_1^2 \int_{-1}^1 (3y^2 - x^2 + 2)\,dx\,dy$$

$$= \int_1^2 \left[3y^2x - \frac{x^3}{3} + 2x\right]_{x=-1}^{1} dy$$

$$= \int_1^2 \left[\left(3y^2 - \frac{1}{3} + 2\right) - \left(-3y^2 + \frac{1}{3} - 2\right)\right] dy$$

$$= \int_1^2 \left(6y^2 + \underbrace{4 - \frac{2}{3}}_{10/3}\right) dy$$

$$= 2y^3 + \frac{10}{3}y \Big|_{y=1}^{2}$$

$$= \left(16 + \frac{20}{3}\right) - \left(2 + \frac{10}{3}\right)$$

$$= 14 + \frac{10}{3} = \frac{52}{3} \blacksquare$$

$$V = \int_{-1}^1 \int_1^2 (3y^2 - x^2 + 2)\,dy\,dx$$

$$= \int_{-1}^1 \left[y^3 - x^2y + 2y\right]_{y=1}^{2} dx$$

$$= \int_{-1}^1 \left[(8 - 2x^2 + 4) - (1 - x^2 + 2)\right] dx$$

$$= \int_{-1}^1 (-x^2 + 9)\,dx$$

$$= -\frac{x^3}{3} + 9x \Big|_{x=-1}^{1}$$

$$= \left(-\frac{1}{3} + 9\right) - \left(\frac{1}{3} - 9\right)$$

$$= -\frac{2}{3} + 18 = \frac{52}{3} \blacksquare$$

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13/

2. (a) Find and classify the local extrema for the function:

$$f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy.$$

$f_x = 12x - 6x^2 + 6y = 0$

$\underbrace{f_y = 6y + 6x = 0}_{y=-x}$

$\Rightarrow 12x - 6x^2 - 6x = 0$

$6x - 6x^2 = 0$

$6x(1-x) = 0$

$\boxed{x=0}$ $\downarrow$ $\boxed{y=0}$ $\quad$ $\boxed{x=1}$ $\downarrow$ $\boxed{y=-1}$

So; $(0,0)$ and $(1,-1)$ are CP's.

$A = f_{xx} = 12 - 12x$

$C = f_{yy} = 6$

$B = f_{xy} = 6$

$(0,0)$: $AC - B^2 = 6(12-12x) - 36 > 0$

$A = 12 > 0$

$\Rightarrow$ local minimum.

value is $f(0,0) = 0$.

$(1,-1)$: $AC - B^2 = -36 < 0$

$\Rightarrow$ saddle point.

12/

(b) Use Lagrange multipliers method to maximize $f(x,y) = xy$ subject to $x + 3y = 120$.

$F(x,y,\lambda) = xy + \lambda(x + 3y - 120)$

$F_x = y + \lambda = 0$

$F_y = x + 3\lambda = 0$

$-y = -\frac{x}{3} \Rightarrow x = 3y$

$F_\lambda = x + 3y - 120 = 0$

$3y + 3y - 120 = 0$

$6y = 120$

$y = 20 \Rightarrow x = 60, \quad \lambda = -20$

$f(x,y) = 60 \cdot 20 = 1200$

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3. Evaluate the indicated integrals:

12/ (a) $\int_{-1}^{1}\int_{0}^{2}(6x^2y-2x)\,dydx. = \int_{-1}^{1}\left[3x^2y^2-2xy\right]_{y=0}^{2}dx$

$= \int_{-1}^{1}(12x^2-4x)\,dx$

$= 4x^3-2x^2\Big|_{x=-1}^{1} = (4-2)-(-4-2) = 8$ ■

13/ (b) $\int_{0}^{1}\int_{0}^{3}e^{x}y\,dxdy = \int_{0}^{1}\left[e^{x}y\right]_{x=0}^{3}dy$

$= \int_{0}^{1}(e^{3}y-y)\,dy$

$= \frac{e^3}{2}y^2-\frac{y^2}{2}\Big|_{y=0}^{1} = \left(\frac{e^3}{2}-\frac{1}{2}\right) = \frac{e^3-1}{2}$ ■

4. The region R is bounded by $x = 3y$, $x = y^2$.

10/ (a) Graph the region R.

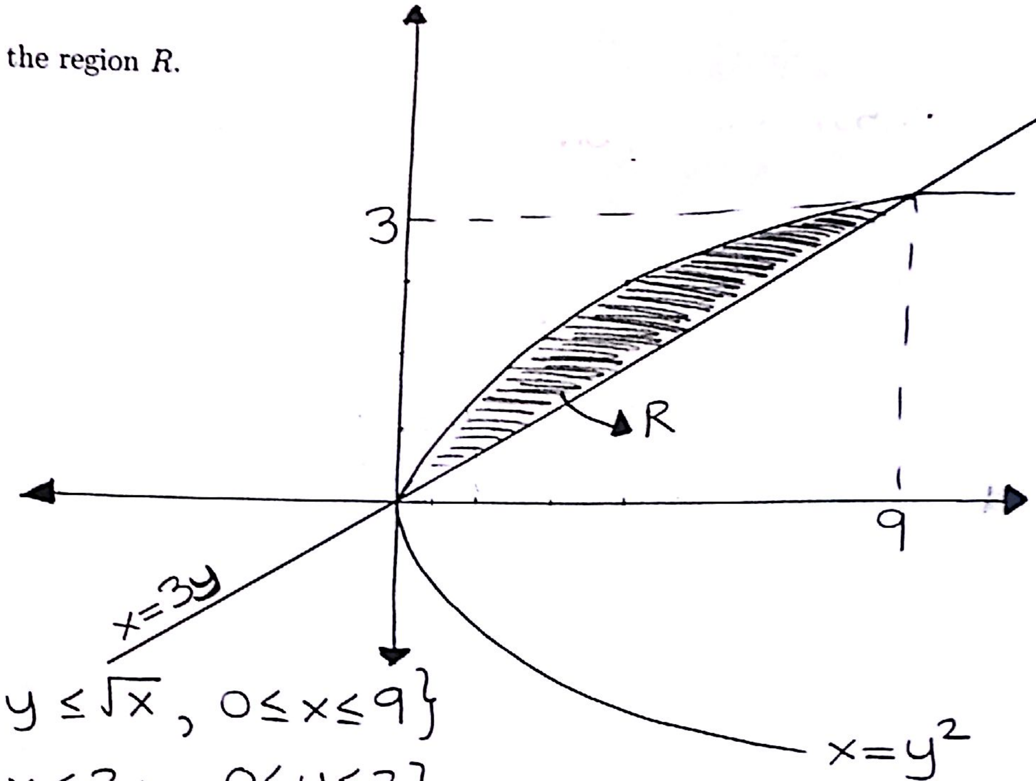
$3y = y^2$

$y^2 - 3y = 0$

$y(y-3) = 0$

$\boxed{y=0}$ $\boxed{y=3}$

$\boxed{x=0}$ $\boxed{x=9}$



$R = \{(x,y) \mid \frac{x}{3} \le y \le \sqrt{x},\ 0 \le x \le 9\}$

$R = \{(x,y) \mid y^2 \le x \le 3y,\ 0 \le y \le 3\}$

15/ (b) Evaluate:

$$\iint_R (x+y)\, dA.$$

$$\int_0^9 \int_{\frac{x}{3}}^{\sqrt{x}} (x+y)\,dy\,dx = \int_0^9 xy + \frac{y^2}{2} \Big|_{\frac{x}{3}}^{\sqrt{x}} dx$$

$$= \int_0^9 \left(x^{3/2} + \frac{x}{2}\right) - \left(\frac{x^2}{3} + \frac{x^2}{18}\right) dx$$

$$= \frac{2}{5} x^{5/2} + \frac{x^2}{4} - \frac{7x^3}{54} \Big|_0^9$$

$$= \frac{2}{5} 3^5 + \frac{81}{4} - \frac{7 \cdot 9^3}{54}$$

not needed

$= 97.2 + 20.25 - 94.5$

$= \underline{22.95}$

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$$\int_0^3 \int_{y^2}^{3y} (x+y)\,dx\,dy = \int_0^3 \frac{x^2}{2} + xy \Big|_{y^2}^{3y} dy$$

$$= \int_0^3 \left(\frac{9y^2}{2} + 3y^2\right) - \left(\frac{y^4}{2} + y^3\right) dy$$

$$= \int_0^3 \left(\frac{15}{2} y^2 - \frac{y^4}{2} - y^3\right) dy$$

$$= \frac{5}{2} y^3 - \frac{y^5}{10} - \frac{y^4}{4} \Big|_0^3$$

$$= \frac{5 \cdot 27}{2} - \frac{3^5}{10} - \frac{81}{4} = \frac{135}{2} - \frac{243}{10} - \frac{81}{4}$$

not needed.

$= 67.5 - 24.3 - 20.25 = \underline{22.95}$

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