25 points	25 points	25 points	25 points	100 points
1	2	3	4	**Total**

MATH 102 CALCULUS II 03.04.2015

İzmir University of Economics Faculty of Arts and Sciences, Department of Mathematics

FIRST MIDTERM EXAM

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1. Evaluate the given integrals:

7/ (a) $\int_1^2 e^x - \frac{4}{x}\,dx$

$$= \int_1^2 e^x dx - \int_1^2 \frac{4}{x} dx = e^x - 4\ln x \Big|_1^2$$

$$= (e^2 - 4\ln 2) - (e - 4\ln 1)$$

($\ln 1 = 0$)

$$= \boxed{e^2 - e - 4\ln 2}$$

8/ (b) $\int_1^3 \left(\frac{-2}{\sqrt{x}} + 3\sqrt{x}\right) dx$

$$= \int_1^3 \left(-2x^{-1/2} + 3x^{1/2}\right) dx = -2\frac{x^{1/2}}{1/2} + 3\frac{x^{3/2}}{3/2} \Big|_1^3$$

$$= -4\sqrt{x} + 2x\sqrt{x} \Big|_1^3$$

$$= (-4\sqrt{3} + 6\sqrt{3}) - (-4 + 2)$$

$$= \boxed{2\sqrt{3} + 2}$$

10/ (c) $\int \frac{20}{x^2 - 25}\,dx \rightarrow \frac{20}{x^2-25} = \frac{A}{x-5} + \frac{B}{x+5}$

$AX + 5A + BX - 5B = 20$

$A + B = 0$

$A - B = 4$

$\boxed{A = 2}$ $\boxed{B = -2}$

So;

$$= \int \frac{2}{x-5} dx - \int \frac{2}{x+5} dx$$

$$= 2\ln|x-5| - 2\ln|x+5|$$

$$= \boxed{2\ln\left|\frac{x-5}{x+5}\right| + C}$$ ⊠

13/ 2. (a) Find the particular solution for the differential equation:

$\frac{dy}{dx} = xe^{3x^2}$; $y(0) = 1$.

$dy = x e^{3x^2} dx$

$\int dy = \int x e^{3x^2} dx \qquad u = 3x^2$

$du = 6x\,dx \rightarrow x\,dx = \frac{du}{6}$

$\mapsto \quad y = \frac{1}{6}\int e^u du$

$y = \frac{1}{6} e^u + C = \frac{1}{6} e^{3x^2} + C$

$y(0) = \frac{1}{6} e^0 + C = 1 \rightarrow \frac{1}{6} + C = 1 \rightarrow \boxed{C = \frac{5}{6}}$.

$$\boxed{y(x) = \frac{e^{3x^2} + 5}{6}}$$

12/ (b) Given the equation of the marginal cost function, $C'(x) = 300 - 0.4x$, find the cost function $C(x)$ if $C(0) = \$500$

$C(x) = \int [300 - (0.4)x]\,dx$

$= 300x - (0.2)x^2 + K$

$C(0) = K = 500$.

So,

$$\boxed{C(x) = 300x - (0.2)x^2 + 500}$$

3. Evaluate the following integrals:

7/ (a) $\int x^2(4x^3-5)^8\,dx$

$u = 4x^3-5$

$du = 12x^2dx \rightarrow x^2dx = \frac{du}{12}$.]

$= \frac{1}{12}\int u^8du = \frac{1}{12}\frac{u^9}{9} + C$

$= \boxed{\frac{(4x^3-5)^9}{108} + C}$ ☒

9/ (b) $\int_2^4 x^2 \ln x\,dx$ $u = \ln x \rightarrow du = \frac{dx}{x}$

$dv = x^2dx \rightarrow v = \frac{x^3}{3}$.]

[$\int u\,dv = uv - \int v\,du$

$= \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3}\cdot\frac{dx}{x} = \ln x \cdot \frac{x^3}{3} - \int \frac{x^2}{3}dx$

$= \ln x \cdot \frac{x^3}{3} - \frac{x^3}{9}\Big|_2^4$

yeterli → $= \left(\ln 4 \cdot \frac{64}{3} - \frac{64}{9}\right) - \left(\ln 2 \cdot \frac{8}{3} - \frac{8}{9}\right)$ ☒

9/ (c) $\int_0^3 xe^{7x}\,dx$ $u = x \rightarrow du = dx$

$dv = e^{7x}dx \rightarrow v = \frac{e^{7x}}{7}$]

[$\int u\,dv = uv - \int v\,du = x\cdot\frac{e^{7x}}{7} - \int \frac{e^{7x}}{7}\cdot dx$

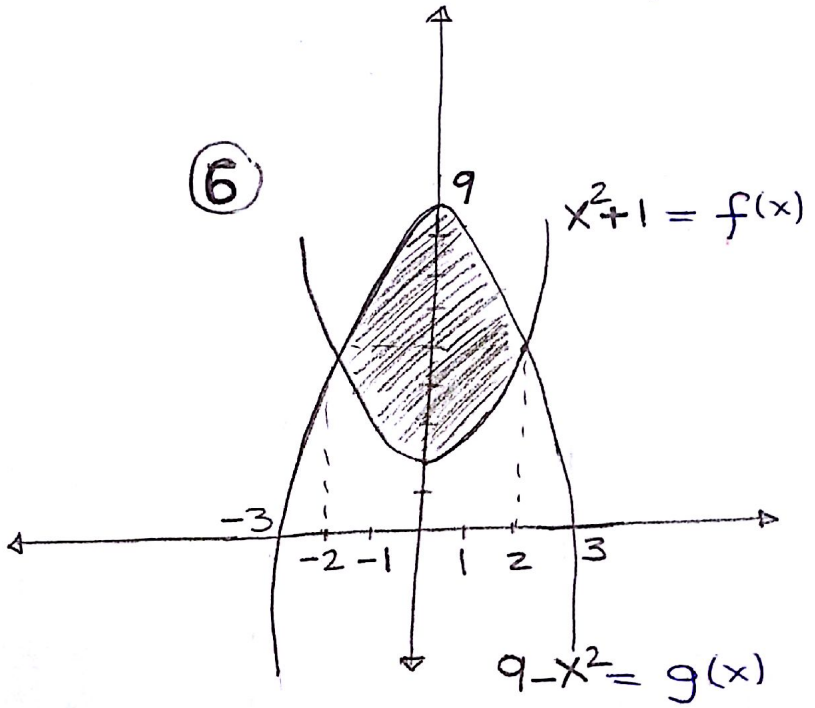
$= \frac{xe^{7x}}{7} - \frac{e^{7x}}{49}\Big|_0^3$

$= \left(\frac{3e^{21}}{7} - \frac{e^{21}}{49}\right) - \left(-\frac{1}{49}\right) = \boxed{\frac{20e^{21}+1}{49}}$ ☒

4. $f(x) = x^2 + 1$ and $g(x) = 9 - x^2$ are given.

10/

(a) Graph f and g on the same coordinate system.



(6)

$x^2+1 = 9-x^2$

$2x^2 = 8$

$x^2 = 4$

$\boxed{x = \pm 2}$

15/

(b) Find the area bounded by f and g.

$$A = \int_a^b [g(x) - f(x)]\,dx$$

$$= \int_{-2}^{2} [(9-x^2) - (x^2+1)]\,dx = \int_{-2}^{2} (8 - 2x^2)\,dx$$

$$= 8x - \frac{2}{3}x^3 \Big|_{-2}^{2}$$

$$= \left(16 - \frac{16}{3}\right) - \left(-16 + \frac{16}{3}\right) = 32 - \frac{32}{3}$$

$$= \boxed{\frac{64}{3}}$$