

= KEY =

25 points	25 points	25 points	25 points	100 points
1	2	3	4	Total

MATH 102 CALCULUS II

07.06.2013

İzmir University of Economics Faculty of Arts and Sciences, Department of Mathematics

FINAL EXAM

Student Name and Department:

Section: Check for your instructor and course program below:

- İbrahim Çanak, Thursday 8:30-11:20
- İbrahim Çanak, Friday 8:30-11:20
- İbrahim Çanak, Friday 12:30-15:20
- Ebru Özbilge, Wednesday, 14:30-17:20
- Ebru Özbilge, Thursday, 8:30-11:20
- Ebru Özbilge, Thursday, 13:30-16:20
- Ahmet Genç, Friday, 08:30-11:20
- Ahmet Genç, Friday, 12:30-15:20
- Sinan Kapçak, Tuesday, 08:30-11:20
- Sinan Kapçak, Thursday, 08:30-11:20
- Sinan Kapçak, Thursday, 13:30-16:20
- Ash Güldürdek, Monday, 13:30-16:20

1. (a) If the marginal profit for producing t units per year is given by:

$$P'(t) = 2t - te^{-t}, P(0) = 0$$

where $P(t)$ is the profit in dollars. find the profit function $P(t)$.

$$P(t) = \int (2t - te^{-t}) dt$$

$$P(t) = t^2 - \int te^{-t} dt = t^2 - [t \cdot e^{-t} + \int e^{-t} dt]$$

$$\begin{aligned} u = t &\rightarrow du = dt \\ dv = e^{-t} dt &\rightarrow v = -e^{-t} \end{aligned}$$

$$P(t) = t^2 + te^{-t} + e^{-t} + C$$

$$P(0) = 1 + C = 0 \Rightarrow C = -1.$$

$$\Rightarrow P(t) = t^2 + te^{-t} + e^{-t} - 1 \quad \square$$

- (b) Evaluate

$$\int_0^1 \int_0^{1-x^2} x \, dy \, dx = \int_0^1 xy \Big|_0^{1-x^2} dx$$

$$= \int_0^1 x(1-x^2) dx$$

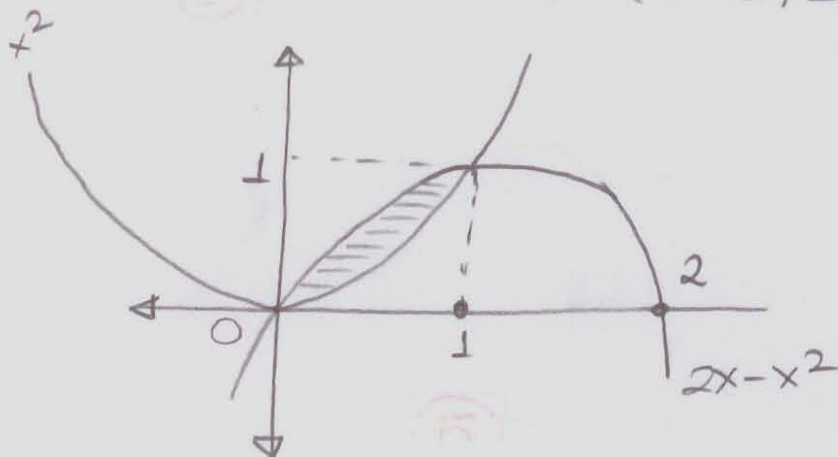
$$= \int_0^1 x - x^3 dx = \frac{x^2}{2} - \frac{x^4}{4} \Big|_0^1$$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \quad \square$$

2. The functions $f(x) = x^2$ and $g(x) = 2x - x^2$ are given.

(a) Graph f and g on the same coordinate system.

$$x^2 = 2x - x^2 \Rightarrow 2x^2 - 2x = 0$$
$$2x(x-1) = 0 \Rightarrow \boxed{x=0} \text{ \& \ } \boxed{x=1}$$



$$2x - x^2 = 0$$
$$x(2-x) = 0$$
$$\underline{x=0} \quad \underline{x=2}$$

(b) Find the area bounded by f and g .

$$A = \int_0^1 [(2x - x^2) - x^2] dx$$

$$= \int_0^1 2x - 2x^2 dx$$

$$= x^2 - \frac{2}{3}x^3 \Big|_0^1 = 1 - \frac{2}{3} = \frac{1}{3} \quad \square$$

3. (a) $f(x, y) = (x^3 + 2y)^4$ is given. Find f_{xy}

$$\begin{aligned} f_x &= 4 \cdot (x^3 + 2y)^3 \cdot 3x^2 \\ &= 12x^2 (x^3 + 2y)^3 \end{aligned}$$

$$\begin{aligned} \Rightarrow f_{xy} &= 12x^2 \cdot 3 \cdot (x^3 + 2y)^2 \cdot 2 \\ &= 72x^2 (x^3 + 2y)^2 \quad \square \end{aligned}$$

(b) Find the volume of the solid under the graph of $f(x, y) = 2x + 3y + \frac{15}{2}$ over the rectangle $R = \{(x, y) \mid -1 \leq x \leq 2, -1 \leq y \leq 1\}$.

$$\begin{aligned} V &= \int_{-1}^1 \int_{-1}^2 (2x + 3y + \frac{15}{2}) dx dy \\ &= \int_{-1}^1 \left(x^2 + 3yx + \frac{15}{2}x \right) \Big|_{-1}^2 dy \\ &= \int_{-1}^1 (4 + 6y + 15) - (1 - 3y - \frac{15}{2}) dy \\ &= \int_{-1}^1 9y + \frac{51}{2} dy \\ &= \frac{9}{2}y^2 + \frac{51}{2}y \Big|_{-1}^1 \\ &= \left(\frac{9}{2} + \frac{51}{2} \right) - \left(\frac{9}{2} - \frac{51}{2} \right) \\ &= 51 \quad \square \end{aligned}$$

$$\begin{aligned} V &= \int_{-1}^2 \int_{-1}^1 (2x + 3y + \frac{15}{2}) dy dx \\ &= \int_{-1}^2 \left(2xy + \frac{3}{2}y^2 + \frac{15}{2}y \right) \Big|_{-1}^1 dx \\ &= \int_{-1}^2 (2x + \frac{3}{2} + \frac{15}{2}) - (-2x + \frac{3}{2} - \frac{15}{2}) dx \\ &= \int_{-1}^2 4x + 15 dx \\ &= 2x^2 + 15x \Big|_{-1}^2 \\ &= (8 + 30) - (2 - 15) \\ &= 51 \quad \square \end{aligned}$$

4. (a) Evaluate

$$\frac{d}{dx}(1 - \cos x) \sin x.$$

$$\begin{aligned} &= \frac{d}{dx} (\sin x - \cos x \cdot \sin x) \\ &= \cos x - [-\sin x \sin x + \cos x \cos x] \\ &= \cos x + \sin^2 x - \cos^2 x \quad \checkmark \\ &\quad \quad \quad -\cos 2x \end{aligned}$$

$$= \cos x - \cos 2x \quad \boxtimes$$

$$\begin{aligned} &= \frac{d}{dx} (1 - \cos x) \sin x \\ &= \sin x \cdot \sin x + (1 - \cos x) \cos x \\ &= \sin^2 x + \cos x - \cos^2 x \\ &= \cos x - \cos 2x \quad \boxtimes \end{aligned}$$

(b) Evaluate

$$\int (5 + \cos x e^{\sin x}) dx = \int 5 dx + \int \cos x e^{\sin x} dx$$

$$= 5x + \int \cos x e^{\sin x} dx$$

subs: $\sin x = u$
 $\cos x dx = du$

$$= 5x + \int e^u du$$

$$= 5x + e^u + C = 5x + e^{\sin x} + C \quad \boxtimes$$