= KEY =

25 points	25 points	25 points	25 points	100 points
1	2	3	4	**Total**

MATH 102 CALCULUS II 08.04.2013

İzmir University of Economics Faculty of Arts and Sciences, Department of Mathematics

FIRST MIDTERM EXAM

Student Name and Department:..................

Section: **Check for your instructor and course program below:**

- [ ] İbrahim Çanak, Thursday 8:30-11:20
- [ ] İbrahim Çanak, Friday 8:30-11:20
- [ ] İbrahim Çanak, Friday 12:30-15:20
- [ ] Ebru Özbilge, Wednesday, 14:30–17:20
- [ ] Ebru Özbilge, Thursday, 8:30–11:20
- [ ] Ebru Özbilge, Thursday, 13:30–16:20
- [ ] Ahmet Genç, Friday, 08:30–11:20
- [ ] Ahmet Genç, Friday, 12:30–15:20
- [ ] Sinan Kapçak, Tuesday, 08:30–11:20
- [ ] Sinan Kapçak, Thursday, 08:30–11:20
- [ ] Sinan Kapçak, Thursday, 13:30–16:20
- [ ] Aslı Güldürdek, Monday, 13:30–16:20

1. (a) Calculate the definite integral:

$$\int_0^1 2x\sqrt{x^2+8}\,dx.$$

$u = x^2+8$ $\quad$ $x=0 \mapsto u=8$

$du = 2x\,dx$ $\quad$ $x=1 \mapsto u=9$

$$\Rightarrow \int_0^1 2x\sqrt{x^2+8}\,dx = \int_8^9 \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big|_8^9$$

$$= \frac{2}{3}\left[9^{3/2} - 8^{3/2}\right].$$

$$= \boxed{\frac{2}{3}\left(27 - 8^{3/2}\right)}.$$

(b) Find the particular solution for the differential equation:

$$\frac{dy}{dx} = \frac{3}{1+x}\ ;\ y(0) = 3.$$

$$dy = \frac{3}{1+x}\,dx$$

$$\int dy = \int \frac{3}{1+x}\,dx = 3\int \frac{dx}{1+x}$$

$$y = 3\ln|1+x| + C \quad \longleftarrow y(0)=3$$

$$3 = 3.\ln 1 + C$$

$$C = 3$$

$$\Rightarrow \boxed{y = 3\ln|1+x| + 3}$$

2. (a) Find the average value of the function:
$f(x) = 3x^2 - 2x + 1$ over the interval $[-1, 2]$.

$$\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2-(-1)}\int_{-1}^{2}(3x^2-2x+1)\,dx$$

$$= \frac{1}{3}\int_{-1}^{2}(3x^2-2x+1)\,dx$$

$$= \frac{1}{3}\left(x^3-x^2+x\Big|_{-1}^{2}\right)$$

$$= \frac{1}{3}\Big[(8-4+2)-(-1-1-1)\Big] = \frac{1}{3}\left[6+3\right] = \frac{9}{3} = \boxed{3}$$

(b) Evaluate the integral:
$\int 9x^2 e^{x^3}\,dx.$

$u = x^3$
$du = 3x^2\,dx \mapsto 9x^2dx = 3du.$

$$\Rightarrow \int 9x^2 e^{x^3}dx = \int e^u 3.\,du = 3\int e^u\,du$$

$$= 3e^u + C$$

$$= \boxed{3e^{x^3} + C}$$

3. (a) The functions $f(x) = x^2 + 1$ and $g(x) = x + 3$ are given.

$f = x^2 + 1 \rightarrow$ parabola

$g = x + 3 \rightarrow$ line.

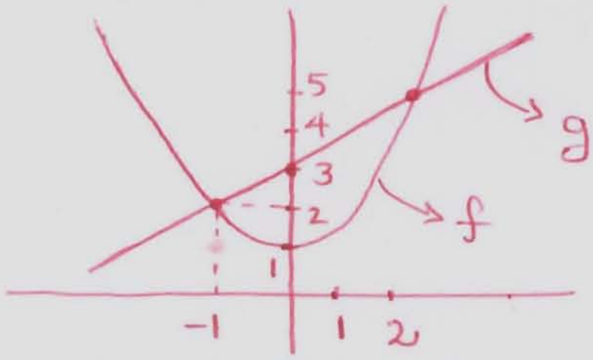
$i-$ Graph f and g on the same coordinate system.

intersection ?

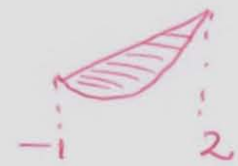
$$x^2 + 1 = x + 3$$

$$x^2 - x - 2 = 0 \quad \mapsto \quad (x-2)(x+1) = 0$$

$$\boxed{x = 2} \quad \boxed{x = -1}$$



Area:



$ii-$ Find the area bounded by f and g.

$$A = \int_{-1}^{2} (g - f)\, dx = \int_{-1}^{2} [(x+3) - (x^2+1)]\, dx$$

$$= \int_{-1}^{2} (x + 3 - x^2 - 1)\, dx = \int_{-1}^{2} (-x^2 + x + 2)\, dx$$

$$= -\frac{x^3}{3} + \frac{x^2}{2} + 2x \Big|_{-1}^{2} = \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$

$$= -\frac{8}{3} + 6 - \frac{1}{3} - \frac{1}{2} + 2 = \boxed{\frac{9}{2}}$$

(b) Find the cost function $C(x)$ when the marginal cost is $C'(x) = 500 - \frac{x}{3}$ and $C(0) = 0$ is given.

$$C(x) = \int C'(x)\, dx$$

$$= \int \left(500 - \frac{x}{3}\right) dx = 500x - \frac{x^2}{6} + K$$

integration constant.

$$C(0) = 0$$

$$\mapsto \quad 0 = K$$

$$\mapsto \quad \boxed{C(x) = 500x - \frac{x^2}{6}}$$

4. Evaluate the following integrals:

(a) $\int \frac{6}{x^2-9}dx = \int \frac{6}{(x-3)(x+3)}dx = \int \frac{A}{x-3}dx + \int \frac{B}{x+3}dx$

$\frac{6}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3} \mapsto AX+3A+BX-3B=6$

$A+B=0$
$A-B=2$
$2A=2 \rightarrow \boxed{A=1}, \boxed{B=-1}$

$\mapsto \int \frac{dx}{x-3} - \int \frac{dx}{x+3} = \ln|x-3| - \ln|x+3| + C$

$= \boxed{\ln\left|\frac{x-3}{x+3}\right| + C}.$

(b) $\int \frac{x+1}{x^2+2x}dx$ $\quad u = x^2+2x$
$du = 2x+2\,dx = 2(x+1)\,dx$

$\int \frac{du/2}{u} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$

$= \boxed{\frac{1}{2}\ln|x^2+2x| + C}$

(c) $\int_1^2 x^4 \ln x\, dx.$ IBP $\quad u = \ln x \rightarrow du = dx/x$
$dv = x^4 dx \rightarrow v = x^5/5$

$\Rightarrow uv - \int v\,du = \ln x \cdot \frac{x^5}{5}\Big|_1^2 - \int_1^2 \frac{x^5}{5}\frac{dx}{x}$

$= \left(\ln 2 \cdot \frac{32}{5} - 0\right) - \int_1^2 \frac{x^4}{5}dx$

$= \frac{32}{5}\ln 2 - \left(\frac{x^5}{25}\Big|_1^2\right) = \frac{32\ln 2}{5} - \left[\frac{32}{25} - \frac{1}{25}\right]$

$= \frac{32\ln 2}{5} - \frac{31}{25} = \boxed{\frac{160\ln 2 - 31}{25}}$