= KEY =

25 points	25 points	25 points	25 points	100 points
1	2	3	4	**Total**

MATH 102 CALCULUS II 11.05.2013

İzmir University of Economics Faculty of Arts and Sciences, Department of Mathematics

SECOND MIDTERM EXAM

Student Name and Department: .....

Section: Check for your instructor and course program below:

- ☐ İbrahim Çanak, Thursday 8:30-11:20
- ☐ İbrahim Çanak, Friday 8:30-11:20
- ☐ İbrahim Çanak, Friday 12:30-15:20
- ☐ Ebru Özbilge, Wednesday, 14:30–17:20
- ☐ Ebru Özbilge, Thursday, 8:30–11:20
- ☐ Ebru Özbilge, Thursday, 13:30–16:20
- ☐ Ahmet Genç, Friday, 08:30–11:20
- ☐ Ahmet Genç, Friday, 12:30–15:20
- ☐ Sinan Kapçak, Tuesday, 08:30–11:20
- ☐ Sinan Kapçak, Thursday, 08:30–11:20
- ☐ Sinan Kapçak, Thursday, 13:30–16:20
- ☐ Aslı Güldürdek, Monday, 13:30–16:20

1. The daily cost function for a company that produces pencils is given by $C(p, w) = 150p + 325w + 1500$ where p and w are the number of plastic and wooden pencils produced each day, respectively.

(a) Evaluate $C(5, 3)$.

$C(5,3) = 150 \cdot 5 + 325 \cdot 3 + 1500$

$= 3225$ ☒

(b) Find:

$i-$ $C_p(p, w)$

$C_p(p,w) = 150$ ☒

$ii-$ $C_w(p, w)$

$C_w(p,w) = 325$ ☒

2. (a) Find the local extrema for the function:

$f(x, y) = 9 - 2x + 8y - x^2 - 4y^2$.

$f_x = -2 - 2x = 0 \rightarrow x = -1$

$f_y = 8 - 8y = 0 \rightarrow y = 1$

$(x, y) = (-1, 1)$.

$A = f_{xx} = -2$

$B = f_{xy} = 0$

$C = f_{yy} = -8$

$AC - B^2 = (-2)(-8) - 0 = 16 > 0$.

$A = -2 < 0$.

$f(-1, 1) = 9 + 2 + 8 - 1 - 4$

$= 14$ is a local max. ⊠

(b) Use Lagrange multipliers method to maximize $f(x, y) = 5xy$ subject to $x + y = 6$.

$F(x, y, \lambda) = 5xy + \lambda(x + y - 6)$.

$F_x = 5y + \lambda = 0 \rightarrow \lambda = -5y$

$F_y = 5x + \lambda = 0 \rightarrow \lambda = -5x$

$-5y = -5x \rightarrow \underline{x = y}$.

$F_\lambda = x + y - 6 = 0 \rightarrow \underline{x + y = 6}$.

$x + y = 6$, $x = y$ $\rightarrow$ $x = 3$, $y = 3$

$(x, y) = (3, 3)$.

$f(3, 3) = 5.3.3$

$= 45$ ⊠

3. Evaluate the indicated integrals:

(a) $\int_0^3 \int_0^2 (x^4 y + y) dx dy.$

$$= \int_0^3 \left[\frac{x^5}{5} y + yx \right]_0^2 dy$$

$$= \int_0^3 \left(\frac{32}{5} y + 2y \right) dy$$

$$= \int_0^3 \frac{42}{5} y \, dy = \frac{\overset{21}{\cancel{42}}}{5} \frac{y^2}{\cancel{2}} \Big|_0^3 = \frac{21}{5} \cdot 9 = \frac{189}{5}$$ ☒

(b) $\int_0^1 \int_0^1 y e^x dx dy.$

$$= \int_0^1 \left[y e^x \right]_0^1 dy$$

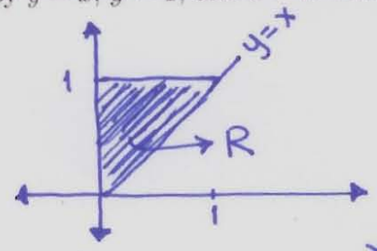
$$= \int_0^1 (y \cdot e - y) dy$$

$$= \frac{e y^2}{2} - \frac{y^2}{2} \Big|_0^1$$

$$= \frac{e}{2} - \frac{1}{2} = \frac{e-1}{2}$$ ☒

4. The region R is bounded by $y = x$, $y = 1$, and $x = 0$. Evaluate:

$$\int\int_R (x + y)dxdy.$$



y-regular

$$= \int_0^1 \int_0^y (x+y)\,dx\,dy$$

$$= \int_0^1 \left[\frac{x^2}{2} + xy\right]_0^y dy$$

$$= \int_0^1 \left(\frac{y^2}{2} + y^2\right) dy$$

$$= \frac{y^3}{6} + \frac{y^3}{3} \Big|_0^1$$

$$= \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$ ☒

x-regular

$$= \int_0^1 \int_x^1 (x+y)\,dy\,dx$$

$$= \int_0^1 \left[xy + \frac{y^2}{2}\right]_x^1 dx$$

$$= \int_0^1 \left(x + \frac{1}{2}\right) - \left(x^2 + \frac{x^2}{2}\right)$$

$$= \frac{x^2}{2} + \frac{x}{2} - \frac{x^3}{3} - \frac{x^3}{6} \Big|_0^1$$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$ ☒