The Mean Residual Life Function of a k-out-of-n Structure at the System Level

Majid Asadi and Ismihan Bayramoglu

Abstract—**In the study of the reliability of technical systems, k-out-of-n systems play an important role. In the present paper, we consider a k-out-of-n system consisting of n identical components with independent lifetimes having a common distribution function F. Under the condition that, at time t, all the components of the system are working, we propose a new definition for the mean residual life (MRL) function of the system, and obtain several properties of that system.**

Index Terms—**Characterization, generalized Pareto distributions, increasing failure rate distributions, mean residual lifetime, order statistics, parallel systems.**

ACRONYM[1]

MRL	mean residual life
GPD	generalized Pareto distribution
IFR	increasing failure (hazard) rate
DFR	decreasing failure (hazard) rate

NOTATION

$m(t)$	MRL function
$T_{k:n}$	life time of the $(n-k+1)$-out-of-n system
$H_n^k(t)$	MRL of the $(n-k+1)$-out-of-n system
$\bar{F}(t)$	survival function

I. INTRODUCTION

An important method for improving the reliability of a system is to build redundancy into it. A common structure of redundancy is the k-out-of-n system. A k-out-of-n system consists of n components, and functions iff at least k of the components function. In the case where $k = 1$, the system is a parallel system; and in the case of $k = n$, the system is known as a series system. Let $T_1, \ldots, T_n$ denote the lifetimes of n components connected in a system with a k-out-of-n structure. Assume that T_i are i.i.d. random variables with common continuous distribution function F, and survival function (reliability function) $\bar{F} = 1 - F$. Let also $T_{1:n} \le T_{2:n} \le \ldots \le T_{n:n}$ be the ordered lifetimes of the components. Then $T_{k:n}$, $k = 1, 2 \ldots, n$, represents the lifetime of the $(n-k+1)$-out-of-n system. If we denote the survival function of the system, at time t, by $\bar{S}(t)$, we have

$$\bar{S}(t) = P(T_{k:n} \ge t) = \sum_{i=0}^{k-1} \binom{n}{i} F^i(t) \bar{F}^{n-i}(t), \qquad t > 0.$$

Assuming that each component of the system has survived up to time t, the survival function of $T_i - t$, the residual lifetime of the components, given that $T_i > t$, $i = 1, \ldots, n$ is

$$\bar{F}(x|t) = \frac{\bar{F}(t+x)}{\bar{F}(t)}. \tag{1}$$

This (1) is the corresponding conditional survival function of the components at age t. The mean residual life (MRL) function m of each component is equal to

$$m(t) = E(T_i - t | T_i \ge t) = \int_0^\infty \bar{F}(x|t)dx = \frac{\int_t^\infty \bar{F}(x)dx}{\bar{F}(t)},$$
$$i = 1, 2, \ldots, n.$$

The MRL function $m(t)$ plays an important role in reliability and survival analysis. It is well known that the MRL function $m(t)$ characterizes the distribution function F uniquely; see, for example, Kotz & Shanbhag [8]. In particular, when T_i are non-negative, for $t > 0$,

$$\bar{F}(t) = \frac{m(0)}{m(t)} e^{-\int_0^t \frac{1}{m(x)}dx}.$$

As the lifetime of a $(n-k+1)-$out-of-n system is $T_{k:n}$, the MRL function of a the system is equal to

$$E(T_{k:n} - t | T_{k:n} > t) = \frac{\int_t^\infty \bar{F}_{k:n}(x)dx}{\bar{F}_{k:n}(t)},$$

where $\bar{F}_{k:n}$ denotes the survival function of $T_{k:n}$. Recently, Li & Chen [10] studied the aging properties of the residual life length of a k-out-of-n system with independent (not necessarily identical) components given that the $(n-k)$th failure has occurred at time $t \ge 0$. They have described also the behavior of several classes of life distributions in terms of the monotonicity of the residual life given the time of the $(n-k)$th failure. Belzunce

Manuscript received January 16, 2005; revised September 9, 2005. Associate Editor: M. Zuo.

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Digital Object Identifier 10.1109/TR.2006.874934

[1]The singular and plural of an acronym are always spelled the same.

et al. [5] define new aging classes, and provide characterizations for a nonparametric class of life distributions based on aging, and variability orderings of the residual life of k-out-of-n systems. For further results on behaviors of aging properties based upon the residual life of k-out-of-n systems, see Li & Zuo [11]. Langberg *et al.* [9] give characterizations of nonparametric classes of distributions by the stochastic ordering of the residual life of the k-out-of-n system, given that the $(n-k)$th failure has occurred at different times. There is a close relationship between the concept dealt with in the present paper, and residual life discussed in the papers [5], [9], [10], [11].

Recently Bairamov *et al.* [3], under the condition that none of the components of the system fails at time t, defined the MRL function of a parallel system as

$$M^1_{(n)}(t) = E(T_{n:n} - t | T_{1:n} > t)$$

and obtained several properties of it. They have also shown that, under some regularity conditions, the survival function $\bar{F}$ can be represented as

$$\bar{F}(t) = \exp\left\{ -\frac{1}{n} \int_0^t \frac{1 + \frac{d}{dx} M^1_{(n)}(x)}{M^1_{(n)}(x) - M^1_{(n-1)}(x)} dx \right\},$$

where $M^1_{(n-1)}(t)$ is the MRL of a parallel system having $n-1$ components. Asadi & Bairamov [1] have given an extension of the $M^1_{(n)}(t)$ as

$$M^k_n(t) = E(T_{n:n} - t | T_{k:n} > t), \qquad k = 1, 2, \ldots, n.$$

The $M^k_n(t)$ defined here is in fact the MRL of the system under the condition that at least $n-k+1$, $k = 1, 2, \ldots, n$ components of the system are working, and the other components have already failed. Several properties of $M^r_n(t)$ are studied in [1].

The aim of the present paper is to give an extension of the definition of the MRL function proposed by Bairamov *et al.* [3], in the case where the system has a k-out-of-n structure, and explore some of its properties. In Section II, we consider a k-out-of-n system, and assume that at time t all the components are working. Under this assumption, we propose a MRL function for the system, and obtain some of its properties.

II. The Mean Residual Life Function of a k-Out-of-n System

In this section, we consider a $(n-k+1)$-*out-of-n* system, and assume that the components of the system have independent lifetimes with common distribution function F & survival function (reliability function) $\bar{F} = 1 - F$. We assume that at time $t > 0$, all the components of the system are working, i.e. $T_{1:n} > t$. Therefore, the residual life time of the system is $T_{k:n} - t | T_{1:n} > t$. If S denotes the survival function of this conditional random variable, then it can be shown that, for $x > 0$,

$$\begin{aligned} S_k(x|t) &= P(T_{k:n} > x + t | T_{1:n} > t) \\ &= \sum_{s=0}^{k-1} \binom{n}{s} \left(\frac{\bar{F}(x+t)}{\bar{F}(t)} \right)^{n-s} \left(1 - \frac{\bar{F}(x+t)}{\bar{F}(t)} \right)^s. \end{aligned} \quad (2)$$

Given that all the components of the system are working at time t, we define the MRL function of the system as

$$\begin{aligned} H^k_n(t) &= E(T_{k:n} - t | T_{1:n} > t) \\ &= \int_0^\infty S_k(x|t) dx \\ &= \sum_{s=0}^{k-1} \binom{n}{s} \int_0^\infty \left(\frac{\bar{F}(x+t)}{\bar{F}(t)} \right)^{n-s} \left(1 - \frac{\bar{F}(x+t)}{\bar{F}(t)} \right)^s dx \\ &= \sum_{s=0}^{k-1} \binom{n}{s} \int_t^\infty \left(\frac{\bar{F}(x)}{\bar{F}(t)} \right)^{n-s} \left(1 - \frac{\bar{F}(x)}{\bar{F}(t)} \right)^s dx. \end{aligned} \quad (3)$$

It should be pointed out here that the MRL $H^k_n(t)$ defined above is in fact the MRL of $T_{k:n}$ at the system level.

Remark 1: It should be pointed out here that the distribution of $T_{k:n} - t | T_{1:n} > t$ is in fact the distribution of the kth order statistics of the sample taken from the conditional distribution $T - t | T > t$.

Remark 2: It can be easily seen, using (3), that the MRL function of the system can be represented as

$$H^k_n(t) = \sum_{s=0}^{k-1} \sum_{j=0}^{s} \binom{n}{s} \binom{s}{j} (-1)^j M_{n-s+j}(t)$$

where

$$M_{n-s+j}(t) = \frac{\int_t^\infty \bar{F}^{n-s+j}(x) dx}{\bar{F}^{n-s+j}(t)}$$

denotes the MRL function of a series system consisting of $n - s + j$ components, $j = 0, \ldots, s$, $s = 0, \ldots, k-1$.

In the following example, we obtain the MRL $H^k_n(t)$ for an important family of distributions:

Example 1: Let X be distributed as GPD with survival function

$$\bar{F}(x) = \left(\frac{b}{ax + b} \right)^{\frac{1}{a}+1}, \qquad x > 0,\ b > 0, -1 < a. \quad (4)$$

The GPD, as a family of distributions, includes the exponential distribution when $a \to 0$, the Pareto distribution for $a > 0$, and the power distribution for $-1 < a < 0$. In this case, we have

$$\begin{aligned} M_{n-s+j}(t) &= \frac{\int_t^\infty (ax+b)^{-(1/a+1)(n-s+j)} dx}{(at+b)^{-(1/a+1)(n-s+j)}} \\ &= \frac{at + b}{(a+1)(n-s+j) - a} \end{aligned}$$

Hence, the MRL function of the system is given by

$$H^k_n(t) = \sum_{s=0}^{k-1} \binom{n}{s} \sum_{j=0}^{s} \binom{s}{j} (-1)^j \frac{at + b}{(n-s+j)(a+1) - a},$$

which is a linear function of t. Note that as $a \to 0$, then $H^k_n(t) \to b \sum_{s=0}^{k-1} (1/n - s)$ i.e., the MRL of a system having independent exponential components does not depend on t.

An important question about the MRL function proposed above is whether it characterizes the underlying distribution F uniquely or not. In the following theorem, we show that, when the distribution function F is absolutely continuous, then it can be uniquely determined by $H_n^r(t)$ & $H_{n-1}^{r-1}(t)$.

Theorem 1: Let the components of the system have a common absolutely continuous distribution function F. Let also f, and $\bar{F}$ denote the density, and survival functions corresponding to F, respectively. Then the survival function $\bar{F}$ can be represented in terms of $H_n^r(t)$ & H_{n-1}^{r-1} as

$$\bar{F}(t) = e^{-\frac{1}{n}\int_0^t \eta(x)dx}, \qquad t > 0, r = 1, \ldots, n,$$

where $\eta(x) = (1 + (dH_n^k(x)/dx))/(H_n^k(x) - H_{n-1}^{k-1}(x))$, and we define $H_{n-1}^0(x) = 0$.

Proof: Let us take $\theta_t(x) = \bar{F}(x)/\bar{F}(t)$, for $x > t$. Then, we have

$$\begin{aligned} H_n^k(t) &= \int_t^\infty \sum_{s=0}^{k-1} \binom{n}{s} (\theta_t(x))^{n-s} (1-\theta_t(x))^s \, dx \\ &= \int_t^\infty \left[1 - \sum_{s=k}^{n} \binom{n}{s} (\theta_t(x))^{n-s} (1-\theta_t(x))^s\right] dx \end{aligned}$$

On taking the derivative of $H_n^k(t)$ with respect to t, we get the equation shown at the bottom of the page, where $r(t) = f(t)/\bar{F}(t)$ denotes the hazard rate of F. This implies that $r(t) = \eta(t)/n$, and hence the proof is complete. ■

Remark 3: We have

$$\begin{aligned} &H_n^k(t) - H_{n-1}^{k-1}(t) \\ &= \int_t^\infty \binom{n-1}{k-1} (\theta_t(x))^{n-k+1} (1-\theta_t(x))^{k-1} \, dx \end{aligned}$$

which shows that the denominator of $\eta(t)$, defined in Theorem 1, is always non-negative. Moreover, If the distribution function F is assumed to be strictly increasing, then we have for $t < x$, $\theta_t(x) < 1$. In this case, the denominator of $\eta(t)$ is always positive.

Remark 4: If we assume, for a $(n-k+1)$-out-of-n system having n independent components with a common distribution F, that

$$H_n^k(t) = \frac{(at+b)}{(a+1)} \sum_{s=0}^{k-1} \frac{1}{n-s}$$

then

$$H_n^k(t) - H_{n-1}^{k-1}(t) = \frac{at+b}{n(a+1)}$$

which in turn implies, using the above theorem, that the underlying distribution F is GPD with a survival function of the form (4). Hence we conclude, based on the result of Example 1, that the MRL function of a system is a linear function of time, iff the common distribution is GPD of the form (4).

In reliability theory, and in the modeling and study of the properties of a lifetime random variable, two important concepts which have widely been studied are IFR & DFR.

A distribution function F is said to be IFR (DFR) if the corresponding hazard rate $r(t) = f(t)/\bar{F}(t)$ is an increasing (decreasing) function of t, where f denotes the density function of F.

We refer the reader to Barlow & Proschan [4] for more details on these Concepts, and other classes of life distributions.

In following theorem, we prove a result showing that, when the components of the system have a common IFR (DFR) distribution, then $H_n^k(t)$ is decreasing (increasing) in t.

Theorem 2: If the components of the system have a IFR (DFR) distribution function F, then $H_n^k(t)$ is decreasing (increasing) in t.

Proof: Let $r(t)$ denote the hazard rate of F. Then, $r(t)$ is increasing (decreasing) iff for $x, t > 0(\bar{F}(x+t)/\bar{F}(t))$ is decreasing (increasing) in t. From this result, it can be easily seen that the survival function $S_k(x|t)$ defined in (2) is decreasing (increasing) in t. This in turn implies that $H_n^k(t)$ is decreasing (increasing) in t, and the proof is complete. ■

The following example gives an important application of the above theorem

Example 2: Let the components of the system have Weibull distribution with survival function

$$\bar{F}(t) = e^{-\left(\frac{t}{\beta}\right)^\alpha}, \qquad t > 0,\ \alpha > 0,\ \beta > 0.$$

Then the MRL $H_n^k(t)$ of the system is decreasing for $\alpha > 1$, and is increasing for $\alpha < 1$.

The following theorem gives a comparison between two $(n-k+1)$-out-of-n systems based on their MRL.

Theorem 3: Let S_1, and S_2 be two $(n-k+1)$-out-of-n systems with independent components. Let the components of S_1, and S_2 have the distribution function F, and G; survival functions $\bar{F}$, and $\bar{G}$; and hazard rates r_F, and r_G, respectively. If, for $t > 0$, $r_F(t) \le r_G(t)$, then $H_n^{1k}(t) \ge H_n^{2k}(t)$, where

$$\begin{aligned} \frac{dH_n^k(t)}{dt} &= -1 - r(t)\left[\int_t^\infty \sum_{s=k}^{n} \binom{n}{s} \left((n-s) \times (\theta_t(x))^{n-s} (1-\theta_t(x))^s - s\,(\theta_t(x))^{n-s+1} (1-\theta_t(x))^{s-1}\right)\right] dx \\ &= -1 - nr(t)\left[\sum_{s=k}^{n} \int_t^\infty \binom{n}{s} (\theta_t(x))^{n-s} (1-\theta_t(x))^s \, dx - \sum_{s=k-1}^{n-1} \int_t^\infty \binom{n-1}{s} (\theta_t(x))^{n-s-1} (1-\theta_t(x))^{s-1} \, dx\right] \\ &= -1 + nr(t)\left[H_n^k(t) - H_{n-1}^{k-1}(t)\right], \end{aligned}$$

H_n^{1k}, and H_n^{2k} denote the mean residual lives of S_1, and S_2, respectively.

Proof: The assumption that $r_F(t) \le r_G(t)$ for $t > 0$ implies that for all $0 < t$, x we have

$$\frac{\bar{F}(t+x)}{\bar{F}(t)} \ge \frac{\bar{G}(t+x)}{\bar{G}(t)}.$$

From this inequality, it can be seen that $H_n^{1k}(t) \ge H_n^{2k}(t)$. ■

III. Some Characterization Results

In this section, we prove some characterization results on GPD.

Theorem 4: Let $T_1, \ldots, T_n$ be i.i.d. non-negative random variables with absolutely continuous distribution function F. Let $T_{1:n}, \ldots, T_{n:n}$ denote the order statistics corresponding to T_i, $i = 1, \ldots, n$. Assume that $\theta(x) = m(x)/m(0)$ where m denotes the mean residual life function of F. Then for $t > 0$, and $k = 1, \ldots, n$

$$\frac{T_{k:n} - t}{\theta(t)} | T_{1:n} > t \overset{d}{=} T_{k:n} \tag{5}$$

iff F is GPD, where d stands for equality in distribution.

Proof: The proof of the 'if' part of the theorem is straightforward, and hence is omitted. To prove the 'only if' part of the theorem, let (5) hold. Then for $x > 0$ we have

$$\begin{aligned} G_n^k(t) &= P\left(\frac{T_{k:n} - t}{\theta(t)} > x | T_{1:n} > t\right) \\ &= \sum_{s=0}^{k-1} \binom{n}{s} (\phi_t(x))^{n-s} (1 - \phi_t(x))^s \\ &= P(T_{k:n} > x) \end{aligned} \tag{6}$$

where $\phi_t(x) = \bar{F}(\theta(t)x + t)/\bar{F}(t)$. Note that

$$\phi_t'(x) = \phi_t(x) \left(r(t) - (x\theta'(t) + 1) r\left(\theta(t)x + t\right)\right)$$

where $r(t) = f(t)/\bar{F}(t)$ denotes the hazard rate of F at t. From this result, on taking the differentiating of both sides of (6), we get

$$\begin{aligned} \frac{dG_n^k(t)}{dt} &= n\left(r(t) - (x\theta'(t) + 1) r\left(\theta(t)x + t\right)\right) \\ &\quad \times \left(G_n^k(t) - G_{n-1}^{k-1}(t)\right) = 0. \end{aligned}$$

This implies that, because $G_n^k(t) - G_{n-1}^{k-1}(t) = \binom{n-1}{k-1}(\phi_t(x))^{n-k+1}(1 - \phi_t(x))^{k-1} > 0$,

$$r(t) - (x\theta'(t) + 1)\, r\left(\theta(t)x + t\right) = 0,$$

which in turn implies that

$$\bar{F}\left(\theta(t)x + t\right) = \bar{F}(t)\bar{F}(x), \qquad t > 0,\ x > 0. \tag{7}$$

It is shown by Oakes & Dasu [12] that (7) holds iff F is GPD of the form (4). See also [2] for a proof of Oakes & Dasu's result under some weaker conditions. ■

Remark 5: To prove the 'only if' part of the theorem, one does not actually need to assume that θ is the mean residual life function of F divided by mean. It is enough to assume that $\theta(t)$ is a non-negative differentiable function of t. Then, from (4), we can easily see that $\theta(t)$ is equal to $m(t)/m(0)$.

Theorem 5: Let $T_1, \ldots, T_n$ be i.i.d. random variables with absolutely continuous distribution function F. Then for fixed values of k & n

$$E(T_{k+1:n} - T_{k:n} | T_{1:n} > t) = c$$

iff the underlying distribution F is exponential, where c is a positive constant.

Proof: Note that

$$H_n^{k+1}(t) - H_n^k(t) = \int_t^\infty \binom{n}{k} \left(\frac{\bar{F}(x)}{\bar{F}(t)}\right)^{n-k} \left(1 - \frac{\bar{F}(x)}{\bar{F}(t)}\right)^k dx$$

Hence the assumption that $E(T_{k+1:n} - T_{k:n} | T_{1:n} > t) = c$ implies

$$\binom{n}{k} \int_t^\infty \left(\bar{F}(x)\right)^{n-k} \left(\bar{F}(t) - \bar{F}(x)\right)^k dx = c\left(\bar{F}(t)\right)^n$$

The kth derivative of both sides of this equation with respect to t, after some simplification, gives

$$\int_t^\infty \left(\bar{F}(x)\right)^{n-k} dx = c\left(\bar{F}(t)\right)^{n-k}$$

which in turn implies that F has to be exponential. This completes the proof of the theorem. ■

IV. Regression of Order Statistics

In recent years, regression of order statistics aroused interest of many statisticians. It is well known that the best unbiased predictor for the $X_{k+m:n}$, given $X_{k:m}$, with the respect to the squared-error loss, is E$(X_{k+m:n} \mid X_{k:n})$. Ferguson [7] considered the problem of classifying the distributions by linearity of regression of E$(X_{k+1:n} \mid X_{k:n})$. The problem to characterize the distributions having a linear regression of E$(X_{k+m:n} \mid X_{k:n})$ is completely solved by Dembinska & Wesolowski [6]. In this section, we give a relationship between the mean residual life function of a $(n-k+1)$-out-of-n system, proposed in this paper, and the regression of order statistics.

Let $f_{T_{k:n}|T_{1:n}>x}(y)$ be the pdf of a conditional random variable $T_{k:n}|T_{1:n} > x$, and $f_{T_{k+1:n}|T_{1:n+1}=x}(y)$ be the pdf of the conditional random variable $T_{k+1:n}|T_{1:n+1} = x$.

Theorem 6: $f_{T_{k:n}|T_{1:n}>x}(y) = f_{T_{k+1:n}|T_{1:n+1}=x}(y)$ for any $y > x$.

Proof: If it is known that $T_{1:n} > x$, then it is known that all the n components are alive at time x. So they all have the same i.i.d. conditional distribution. Hence $T_{k:n} \mid T_{1:n} > x$ is the kth order statistic of these i.i.d. random variables. On the other hand, if is known that $T_{1:n+1} = x$, then it is known that all the other n components are alive at time x. Hence $T_{k+1:n+1} \mid T_{1:n+1} = x$ is the kth order statistics of these n other i.i.d. random variables, and therefore it has the same distribution as $T_{k:n} \mid T_{1:n} > x$. Then the result follows. ■

Therefore we can write also: If $H_n^k(t) = E(T_{k:n} - t | T_{1:n} > t)$, and $g_{k+1:n+1}(t) = E\{T_{k+1:n+1} | T_{1:n+1} = t\}$, then $H_n^k(t) = g_{k+1:n+1}(t) - t$.

Corollary 1: The distribution function F can be represented by the regression function $g_{k+1:n+1}(t)$ as

$$\bar{F}(t) = \exp\left\{ -\frac{1}{n} \int_0^t \frac{g'_{k+1:n+1}(x)}{g_{k+1:n+1}(x) - g_{k:n}(x)} dx \right\}.$$

Proof: The proof follows from Theorem 6, and the fact that $H_n^k(t) = g_{k+1:n+1}(t) - t$. ■

ACKNOWLEDGMENT

The authors would like to thank three anonymous referees, the Associate Editor Dr. M. Zuo and the Managing Editor Dr. J. W. Rupe for their valuable comments, which led to the improvement of the presentation of this paper.

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